THE EFFECT OF FERTILIZATION ON STEM CHARACTERISTICS OF POLE STAGE SITKA SPRUCE

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ABSTRACT OF THESIS

The effect of PK fertilization on the stem characteristics of pole stage Sitka spruce (Picea sitchensis (Bong.) Carr.) was examined in this study.

Ring width measurements were taken from discs cut from each mid-intermodal position, up to an overbark diameter of 7 cm, for 40 control and 40 fertilized trees in replicated experiments, and were used for the examination of the internal pattern of growth of the trees. Using computer plotting facilities tree diagrams were constructed showing the different stages of development of the trees. Following fertilization, ring width and ring area increased dramatically over the stem of the fertilized trees, indicating possible changes in the stem form. The effect of fertilization was reflected clearly in the radial ring sequences, the longtitudinal ring sequences and the sequences of rings parallel to the pith, as well as in the "contour diagrams" which were developed in this study.

A method for estimating tree volume was developed which could be used when a factor - such as fertilization - might influence the shape of the stem.

Analysis of the results revealed that there were statistically significant differences between control and fertilized trees in terms of basal area, cross-sectional area at half the total height, total height and volume. Examination of the form factor revealed that the form factor of the fertilized trees increased following fertilization.

Stem form comparisons between control and fertilized trees were carried out using:

1. Principal component analysis, and
2. Relative diameters.

The results of the above comparisons confirmed the form factor changes and indicated that changes occurred in the middle to upper part of the stem. This indicates the need for form measurements to be taken when estimating volume responses to fertilization on an area basis.

A diameter prediction model was developed using the three principal components derived from the principal component analysis and gave good results.

Trees responded to fertilization according to their position in the stand canopy: dominant trees with bigger and more vigorous crowns responded more to fertilization than co-dominant and subdominant trees.

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INTRODUCTION

1.1
GENERAL INTRODUCTION AND SCOPE OF THE STUDY

In recent years there has been a trend for afforestation to concentrate on poorer soils since the land to be used for such purposes has become scarcer. On the other hand, there exists a growing demand for wood products. The above conditions led to the development of new techniques to be applied for an increase in wood production. One of the silvicultural practices by which we attempt to create an environment favourable for tree growth is fertilization. Forest fertilization has long been used for this purpose (Heiberg and White, 1951; Tamm, 1968). The improvement of tree growth by fertilization is achieved by augmenting the natural supply of plant nutrients from the soil in order to correct or forestall deficiencies (Everard, 1974). Fertilization provides the forester with much more control over the growth of his crops, and makes it possible to raise the level of yield without endangering the future yield potential of the forest.

As well as influencing total volume increment, fertilization may affect the distribution of increment along the stem of the trees (Bower, 1973; Brix and Ebell, 1969; Gessel et al, 1969; Mitchel and Kellogg, 1972). Such changes in the distribution of increment may have important consequences in utilization and management. It is well known that a decrease in the taper of the trees may increase the outturn of saw-logs. In some cases there have been reports that stem form of the trees changed markedly following fertilization (Woolons and Will, 1975; Whyte and Mead, 1977). In some other cases slight changes in the taper of the trees were reported (Miller and Cooper, 1973), and finally there were cases in which stem form of the trees remained unaltered (Bower, 1973; Barclay et al, 1982). Table 1, while not being comprehensive, summarises many of the studies looking at the response of fertilization in various species in different countries.

Application of fertilizers may also cause a shift in balance of increment towards the biggest trees in the stand (Mitchell and Kellog, 1972; Windsor and Reines, 1973). Such a change may influence the pattern of shading in the stand and therefore by increasing the suppression of the smaller trees will influence the time of thinnings.

Usually responses of volume or volume increment of trees to fertilization are inferred from measurements of basal area and/or height. Unfortunately, little is known as to how representative such measurements are in relation to the pattern of the response higher up the stem since, as we have seen from the previous cited reports, the stem form of trees may be changed following fertilization.

Assmann (1970) concluded that basal area increment at breast height, which is frequently used as a substitute for volume increment, is a doubtful value because volume increment also depends on height and stem form and thus differs widely. Reukema (1971) reported that the error in estimating volume increment - over a four year period - was 5 to 10 per cent if a 1 per cent change in form factor was not taken into account.

Estimating volume responses to fertilization, Mitchell and Kellogg (1972) concluded that the estimation of volume response in dominant trees from breast height measurements should possibly be approached with caution. Finally, Flewelling and Yong chi Yang (1976), estimating volume responses to fertilization, concluded that the accuracy of volume tables which were used to estimate the volume responses were biased by fertilization.

From all the above cited references it might be concluded that it is difficult to estimate responses in the volume of the trees by measuring their basal areas and heights, since fertilization might

TABLE 1

RESPONSE OF DIFFERENT SPECIES TO FERTILIZATION

SPECIES	AGE(when fertilized)	REFERENCE	YEAR OF REFERENCE	COUNTRY	FERTILIZER	PERIOD EXAMINED (years)	VARIABLES AFFECTED BY FERTILIZATION			
							DB or DBH	Height	Form	Volume
Corsican pine	36	Miller & Cooper	1973	Britain	N	6	+	+	0	+
Loblolly pine	8	Matziris & Zobel	1976	U.S.A	PK	4	+	+		+
Loblolly pine	23	Windsor & Reines	1973	U.S.A	NPK	6	+			
Loblolly pine	24	Wells et al	1976	U.S.A	N	5				+
Radiata pine	13	Wollons & Will	1975	N.Zealand	NP,NPK,N	5	+		+	+
Radiata pine	40	Whyte & Mead	1977	N.Zealand	NP	5	+		+	+
Red pine	30-35	Heiberg et al	1964	U.S.A	K	15	+	+		
Sitka spruce	5	Farell & McAleese	1972	Ireland	N,P	4		+		
Sitka spruce	8	McIntosh	1978	Britain	PK	3		+		
Sitka spruce	11	Davies et al	1971	Britain	P	4	+	+		
Black spruce	65	Weetman	1975	Canada	N	10	+			+
White spruce	35	Gagnon et al	1976	Canada	K	10	+			+
Douglas fir	20	Brix & Ebell	1969	Canada	N	3	+	+		
Douglas fir	24	Barclay et al	1982	Canada	N	9	+	+	0	+
Douglas fir	20-45	Bower	1973	U.S.A	NPK	10	+	+	0	+
Douglas fir	47	Flewelling & Yong chi Yang	1976	U.S.A	N	5				+
Douglas fir	49	Mitchell & Kellogg	1972	Canada	N	5				+

Fertilizers 1. N - Nitrogen 2. P - Phosphorus 3. K - Potassium

Response 1. + - positive response to fertilization
2. 0 - no response to fertilization
3. blank spaces denote that either results are not presented, or not examined

affect the form of the trees. For this reason measurements of the form of the trees should be taken into account.

From the studied bibliography negative results to fertilization do not appear so far. Möller and Rytterstedt (1974), who examined a lot of N fertilization trials in 21 trial areas of Pinus silvestris and Picea abies stands in Sweden, stated that "Although many of the trials covered a period of twelve years following fertilization they still appear to be too young for a reliable answer to the question of possible negative effects to be given. Nonetheless, the mean curves for the latter part of the period would indicate that scarcely any negative effects are to be expected".

The effects of fertilization on the growth of the trees, and the possibility that the redistribution of increment following fertilization might have an influence on the stem form of the trees, were the main reasons for this study.

The following objectives were set:

1. To search for a method for sectional volume estimation which could be used to detect any possible influence of fertilization on the stem form, since volume and stem form are closely related.
2. To examine the influence of fertilization on the internal pattern of growth of the trees, since this might explain possible changes in the external shape of the trees.
3. To examine how trees respond in terms of:

   a) basal area

   b) height

   c) form factor, and

   d) volume

   as well as the duration of the response.
4. To examine the behaviour of response of trees belonging to different dominanceclasses.
5. To analyse the influence of fertilization on the stem

profile of the trees and develop diameter prediction functions.

6. To examine the relationship between volume, volume increment and the basal area of the trees, under the influence of fertilization.

For the fulfilment of these objectives, it was decided, following consultation with the Forestry Commission, that suitable material for the study could be provided by a 4x4 factorial experiment in Glentrool forest in SW Scotland. This experiment was designed to study the effect of top dressing PK fertilization on pole stage Sitka spruce (<u>Picea sitchensis</u> (Bong.) Carr).

1.2 SITKA SPRUCE - FERTILIZATION

Sitka spruce is one of the most important forest species used in afforestation in Britain. In northern England and southern Scotland 240,000 hectares or two thirds of the productive woodland area are spruce, and mainly Sitka spruce (Jeffrey, 1979). It is less widespread in the east and unusual in East Anglia and Yorkshire (Mitchell, 1972). In West Galloway district - where the experimental area is situated - Sitka spruce accounts for about 65% by area of the growing stock, and comprises almost 90% of the current planting programmes (McIntosh, 1978).

The main reason for the expansion of Sitka spruce plantations in Britain is its rapid early growth as well as its greater productivity as compared with other species (Malcolm, 1970). Malcolm, who studied the influence of site factors on the growth of Sitka spruce in different forests in Scotland, pointed out that productivity in each of the examined forests was affected by local climatic effects as well as by the phosphorus status of the site.

Because of the success in the establishment and growth of Sitka spruce in the upland areas, it was planted over a wide range of sites with different fertility. The value of adding phosphatic fertilizers at planting on the poorest sites was soon realised, and with the development of the plantations it became apparent that on poor sites further top dressings with phosphatic fertilizers were needed (McIntosh, 1981).

On peaty ironpan soils the main factors limiting the growth of young Sitka spruce are phosphorus (P) and nitrogen (N) deficiencies, and on deep unflushed peat potassium (K) deficiency is another factor limiting its growth (MacKenzie, 1974). The response of Sitka spruce to phosphorus on mineral soils at planting is currently being examined on over 30 sites, in randomised block experiments (McIntosh, 1981), but the trees are considered too young to yield useful results. According to McIntosh (1981) on shallow peat soils with up to 30cm of peat, as a general rule phosphorus should be applied at planting and for those with over 30cm peat, phosphorus and potassium should be applied. On deep peat (over 45cm peat), phosphorus and potassium should be applied at planting.

Responses in height of pre-canopy closure stage Sitka spruce to PK fertilization over a three year period have been reported by McIntosh (1978). Also responses in height of similar stage Sitka spruce to phosphate and sulphate of ammonia fertilization have been reported by Farrel and McAleese (1972) over a four year period. Davis et al (1971) detected a diameter response in Sitka spruce - eleven years old - following phosphorus fertilization. A short term response to nitrogen fertilization of six year old Sitka spruce in terms of height has been also reported by McIntosh (1983). Dickson and Savill (1974), who examined the relationship between growth of Sitka spruce planted on deep oligotrophic peat, and uptake of P, K and N, found height increases and suggested the use of N fertilizers also, apart from P, for this soil category.

However, more important responses than those of height, such as basal area, volume and form factor responses in older Sitka spruce stands do not seem to have been reported so far.

Since the response to fertilization is expressed in terms of changes in basal area, total height, stem form and volume, the measurements of these variables is important and will be discussed below.

1.3
STEM VOLUME VARIABLES AND THEIR MEASUREMENTS

The common methods adopted in forestry for the volume estimation of the trees include measurements of the following variables:

1. Diameter at breast height (1.3m above the ground level).
2. Total height, and
3. A measure of the stem form (form factor, form quotient).

There is a variety of procedures and a range of instruments for measuring the above variables or their increment.

1.3.1
DIAMETER MEASUREMENTS

When diameter measurements are being taken to estimate responses to a treatment care must be taken for these measurements. The reason for this is that basal area (cross-sectional area at breast height), is based upon these measurements and because it is very well known that basal area and volume are very closely related.

Generally, diameter increment estimation can be obtained by taking repeated diameter measurements and in this case the exact height where the first measurement was taken must be marked as a guide for the future measurements. In regions where tree growth follows a definite annual growth pattern, direct measurements of diameter growth can be obtained by:

1. Taking borings with an increment borer, and
2. Measuring the diameter on cross-sectional cuts (discs).

The accuracy of the diameter increment determination of the individual trees depends on the following factors (Siostrzonek, 1958):

1. The number of measurements (how many radii)
2. The age of the trees (eccentricity increases with age)
3. The choice of the radii
4. The mathematical evaluation (method of averaging)
5. The depth of the boring (error increases with depth)

Measuring diameter increment on discs is comparably more accurate because it permits a direct measurement of the desired radius (i).

According to several investigators (Siostrzonek, 1958; Müller, 1957) trees during their life time pass through various phases of stem cross-section form development. Therefore in each particular case for diameter increment estimation where effort versus accuracy must be balanced, one will have to consider the aforementioned factors for obtaining the desired results. According to Siostrzonek (1958) good results in the estimation of diameter or basal area increment are obtained from measurements on discs when four radius measurements are taken perpendicular to one another (crosswise), with one arm of the cross coinciding with the maximum radius, and the average is obtained with the arithmetical mean. Double the arithmetic mean will give us the desired diameter increment at any time interval of tree life. Finally, planimetering of the discs can be used for the direct estimation of basal area increment with very good results.

The diameter at breast height is usually measured with a diameter tape or a caliper. Use of a diameter tape will always have an over-estimation of the mean diameter, except in the case of truly circular cross sections. When a caliper is used, usually two measurements at right angles are taken and the average is used. In this case some positive bias is introduced (Loetsch et al, 1973). Finally, when optical instruments (prisms, relascope, etc) are used, diameter readings must be taken from more than one direction.

1.3.2
HEIGHT MEASUREMENTS

Total height is considered the vertical distance from the ground to the top of the tree. There is a range of instruments in use for the height measurements of the trees. Altimeters like Haga and Blume-leiss, and optical instruments like the Biterlich relascope. These instruments are used for standing trees. In this case we have an indirect measurement of the tree height from the ground level based on the principles of trigonometry. When these instruments are used there is always the possibility of errors because of wrong adjustment of the instruments, leaning trees and inability to define clearly the top of the tree.

In standing trees height rods can also be used when the trees are young, or rods in combination with climbing the tree and using a tape. One source of errors with rods is the lack of coincidence of the height rod with the top of the tree. This can be overtaken with the use of binoculars from the ground.

Repeated total height measurements at the beginning and the end of the growth period can give the annual height increment. Height increment itself can be easily measured in some coniferous species which produce one branch cluster per year. The height increment in this case can be measured if the termination of previous year growth is marked as a reference point. Elongation of the stem at intervals during the growing season and for the total period can be measured from this reference point. Care must be taken for coniferous species which produce more than one branch cluster per year. For forest species which do not show a reçognisable pattern of height growth, past years' height increment can be obtained by stem analysis.

1.3.3
FORM MEASUREMENTS

The stem form can be defined as the decrease in thickness, in terms of diameter of a tree stem, from the base upwards. Stem taper according to the previous definition is a synonym. It can also be referred to as the relative change in stem diameter with increasing height. Using the above definition stem form can be accurately represented by the stem 'profile' of a tree.

Despite the extreme variability of stem form of the trees which makes it almost impossible to formulate general rules with application to all stems of a species, numerous attempts have been aimed to the expression of the stem form as a mathematical expression. Larson (1963) who studied a lot of early publications concerning the stem form pointed out that: "It can be shown that tree growth and stem form development follow certain general laws or patterns of growth that are inherited. However, these basic patterns can be modified by many environmental factors and by silvicultural practices."

The earliest method of expressing the form of a tree was by the use of form factors. The form factor of a tree is the amount by which the volume of a cylinder of identical basal area and height has to be multiplied to give the actual volume of the tree. Thus form factor can be calculated from the formula:

$$F = \frac{V}{G \times H} \qquad (1)$$

where: F is the form factor

V is the actual volume of the tree

G is the basal area of the tree

H is the height of the tree

This method of expressing the form factor of the tree has the disadvantage that it can not be measured directly on a tree and this was the reason that this method was superceded by the form quotient method in which a lower diameter was measured at a fixed height (1.3m above ground level) and the upper diameter was measured at a variable height. In this way the form quotient varied with the size of the tree. To correct for this, form quotient was later redefined as the ratio between the diameter at half the height of the tree above breast height and the diameter at breast height. The measurement of the upper diameter was still the main difficulty of this method.

Both these indices of stem form are rather superficial expressions. The second method of defining the stem form which superceded the form factor method is a rather unreliable expression of the form, because trees of the same form quotient may have quite different stem forms (Matte, 1949).

Many attempts have been made in the past to express the stem form mathematically in terms of stem curves. Several theories for the development of stem form were advanced (Metzger, 1893; Gray, 1956). Metzger (1893) based his theory of stem form development on the idea that the wind action was the main factor of stem form development and concluded that tree stems ought to conform to the dimensions of a cubic paraboloid in order to withstand the wind forces. One more recent modification of the above theory could be considered Gray's "taper line" theory. Gray (1956) based on the same ideas, that is wind action was the main factor for the development of stem form, pointed out that the quadratic paraboloid was more consistant with the mechanical (bending stresses) requirements of the tree. Gray, with numerous examples of trees from Australia and Great Britain, showed that graphs of d^2 over h (diameter squared over height) gave better fits and over a greater part of the stem than graphs of d^3 over h (Metzger's theory). Gray's "taper line" method, although advantageous when applied on standing trees gives biased results (Carron and McIntyre, 1959). Also according to Newnham (1965) application of this method gave good results for the part of the stem between 15 and about 80 per cent of tree height, although there were certain departures from the taper line, and in the lower part of the stem the diameter was less than the expected from the calculated "taper line".

An extensive review of the literature about the mathematical representation of stem curves is given by Sterba (1980). The most important points that emerge from this review are:

1. The greatest part of the research is directed towards the mathematical formulation of stem curves.
2. The possible application of the stem curves is only mentioned briefly in most of the cases when the objectives are stated or it is suggested as a "further research" at the end of the theoretical work.
3. Only a small part of the research goes into explanation of the stem form, relationships between the shape of the stem curve and the sociological status of the trees in the stand and examination of the shape of the stem curve on silvicultural treatments. Finally, he concluded that further investigation will be worthwhile if the influence of site and silvicultural treatments on the stem form is considered.

Among the other methods described in Sterba's paper, multivariate methods have been used for the stem form description, (Fries and Matern, 1966). Such methods have been also used for comparisons of stem curves between different species (Fries, 1965), and for the same species under different sites and treatments (Liu and Keister, 1978; Mendiboure, 1972).

Use of multivariate methods, such as Principal Component Analysis, might result in more general expression of stem form since such methods provide one with means of considering the general dimensions of variability as a whole. Application of Principal Component Analysis might help in the definition of stem form in order that comparisons between the stem forms of control and fertilized trees could be carried out for tracing probable responses along the stem of the trees and not in terms of form factors only.

Finally, another method that might be of importance for comparison of stem form in cases that this might have been affected by a treatment - such as fertilization - is that of using relative stem curves (Hohenadl 1922, Assmann, 1970). Relative stem curves are stem curves which have been constructed using ratios of diameters at proportional heights (e.g. tenths of total height) to a reference diameter (e.g. the one at the first tenth of total height measured from the ground level) plotted over height. Prodan (1944) who elaborated Hohenadl's method examined various mechanical and physiological theories about the stem curves in order to determine which statistically defined curve corresponds better with these theories, concluded that the relative stem curves correspond better with the physical and mechanical premises than polynomials in general.

1.4 FACTORS AFFECTING STEM FORM

There is evidence that most silvicultural practices which result in an alteration of the growing space of the crown will have an influence in the stem form development (Larson, 1963; Assmann, 1970).

Usually the part of the stem covered by the crown of the tree is strongly tapered (Duff and Nolan, 1957) because of the progressive increase in the number of branches from the apex downwards, and because of the cumulative contribution distribution of these branches to the growth of the stem. Therefore open grown trees possessing longer crowns are more tapered than trees grown in close distances (Belyea, 1925; Braathe, 1953; Jorgensen, 1967). As the tree crown recedes, and the branchless part of the stem elongates with increasing age or stand closure, the stem becomes more cylindrical (Bickerstaff, 1946). This tendency towards cylindricity usually results from a concentration of growth near the position of the crown base (Labyak and Schumacher, 1954; Duff and Nolan, 1957).

The influence of age in stem form was examined by Stoate (1942) who showed that with trees of the same diameter at breast height but different ages the younger trees would have greater taper because, as trees age, diameter increment declines faster than height increment. Reukema (1961) also showed that the position of maximum increment was shifting upwards with increasing age and therefore trees aging were becoming more cylindrical.

As implied above, silvicultural treatments such as thinning, pruning and fertilization can change the growth pattern by introducing artificially new growth conditions in the stands.

Thinning usually influences the stem form by increasing the growing space of the trees in a forest. The main effect of thinning is to convert a stand-grown tree to a tree which grows in more open conditions resulting in a shift of increment down the stem in response to the increased exposure and crown size (Reukema, 1961). The increased wind action which will increase the tree swaying will result in the development of mechanical stress in the lower part of the stem which also probably stimulates increased growth (Metzger, 1893; Gray, 1956; Brix, 1976).

In the literature about stem form studied by Larson (1963), there is evidence that thinning decreases cylindricity and the main factor appears to be the wind action which promotes growth in the lower part of the stem. Larson also cited reports where there was no change in stem form following low intensity thinnings. He also cited evidence that in the best sites heavy thinning did not result in a deterioration of stem form, but the poorer the site the greater the decrease in cylindricity. Finally he stated that many of these contradictions could be accounted for by the type of thinning as well as intensity of thinning and prior stem form.

As expected, pruning has the opposite effect to thinning since by reducing the crown of a tree artificially converts it to a close grown tree. According to Larson (1963) pruning results in a reduction of increment at the stem base and a concentration of increment in the upper stem parts, and therefore tree stem becomes more cylindrical. Fertilization may have a marked effect on stem form in some cases as discussed in section 1.1. It has been suggested that the greater response might be at the base of the crown (Miller and Cooper, 1973). There has been evidence from other experiments (Brix, 1976) that nitrogen fertilization has provided the trees with bigger crowns and more foliage in the first season of application.

It is likely that improvement of tree growth conditions through fertilization might have caused a redistribution of increment over the stem which might introduce greater cylindricity over the stem. It is equally likely that fertilization might cause an advancement of the whole development of the stand by promoting growth conditions and that the response is similar to an advancement in the apparent age of the tree and according to Larson (1963) in this case there might have been an improvement in the stem form since ageing improves cylindricity.

Finally, the influence of these silvicultural treatments on stem form will in turn depend on the influence of heredity and site, and stand structure. Larson (1963) had the opinion that responses in form are not invariable and often changes in form fail to occur following a silvicultural treatment, although in several instances he attributed that to the prior conditions of trees, the experimental techniques and frequently to the analysis and interpretation of the results.

1.5
VOLUME ESTIMATION

Use of one diameter and the height of the tree stem will only provide us with an accurate estimate of the volume of the trees if form factor or stem profile can also be taken into account.

As an alternative the volume of a tree can be calculated by summing the volume of sections of the stem. Unless the sections are very short we must assume that each section corresponds to a solid of revolution (Spurr, 1952; Husch, 1963; Anuchin, 1970). This gives rise to formulae such as those of Huber and Smalian which can be applied to sections of tree stems for volume estimation.

Usually the tree stem is divided into parts of equal length, and at least one diameter measurement is taken for each of them (in accordance with the formula to be applied for the volume estimation). Such diameter measurements might be unrepresentative since they might coincide with whorl nodes, defective positions on the stem, or perhaps with other such irregularities. The numerous points of inflection - sudden drops of diameter that change the shape of the stem - (Grosenbaugh, 1966; Assmann, 1970) might add to that so that the final volume estimate might be inaccurate. If we are interested in the volume responses to some treatment, such as fertilization, then it is possible that these points which were suitable for defining the sections for untreated trees, might be unsuitable for the trees following treatment due to shape changes.

When the volume of the trees is to be estimated there are several methods for obtaining the diameter measurements for this purpose such as:

1. By the use of optical instruments such as prisms, dendrometers (Barr and Stroud) or the Biterlich relascope.
2. By climbing the tree and taking the diameter measurements at different heights with a tape or a caliper.
3. By felling the tree and taking the diameter measurements as before.

The total height is also measured as described in section 1.3.2 using a tape.

Schmid et al (1971) give the sources of error and costs for height and diameter measurements for the volume estimation using various instruments.

For the volume increment estimation of a tree, or when volume responses are to be estimated, repeated diameter and height measurements can be taken at the beginning and the end of a period, using one of the methods described above. As an alternative stem analysis methods could be applied estimating increment, especially when there are no measurements of the volume variables at the start of an experiment.

Application of such a method can yield very accurate results of volume increment under bark, for any year before or during the period of a fertilization experiment. The advantage of this method is its ability for more accurate diameter measurements to be taken since the diameters will be measured on cross-sectional cuts (discs). It also provides one with the ability of checking previous increment trends in the stand.

An accurate volume estimation of a sample of trees is the basis for a better total volume estimation of the population. Therefore the volume estimation of each tree in a sample will have to be estimated as accurately and efficiently as possible. For these reasons and because the usual methodsof volume estimation of the single trees are not very "sensitive" for situations where the growth pattern of the trees has been influenced by a treatment, it is considered necessary to search for a better sectional volume estimation for single trees.

After all the above discussion it can be concluded that the bigger the number of diameters taken along the stem of the trees and measured on cross-sectional cut discs, the better the results of volume estimation. Application of stem analysis techniques enabling annual or periodic volume increment trends during or before the experiment to be estimated, would yield quite accurate results. Since volume and stem form of the trees are very closely related, the same diameter measurements would permit reconstruction of the stem curves at the beginning and the end of a period under examination, so that the influence of fertilization on the stem form of the trees could be revealed all along the stem curve and not only at a standard position (e.g. breast height). Finally, it can be said that the need arises for a better sectional volume estimation method that might cope better with situations that the stem form is influenced by the treatment so that more reliable estimates of stem volume could be obtained.

CHAPTER 2

MATERIAL

2.1 SITE DESCRIPTION

The site is located in Glentrool forest, Galloway, west of Dumfries. The Forestry Commission is conducting various experiments in this forest trying to assess the effects of fertilization on the increment of trees. The study area in this forest was compartment 12 and the species is Sitka spruce (Picea sitchensis (Bong.) Carr.) planted in 1948 (turf planting, no ploughing) at an initial spacing of 1.5x1.5 metres. The elevation in the study area is 122 m with an easterly aspect, exposure is moderate and the annual rainfall is 1750 mm. The underlying geology is predominantly of the Silurian period. The soil is flushed deep peat, (F.C. classification 9b) of 1 m. in depth on the west side, becoming deeper towards the east side of the study area.

The object of this experiment was, according to the Forestry Commission, to observe the effect of P (phosphate) and K (potassium) top dressing on pole stage Sitka spruce. The experimental design includes a 2x2 factorial design in 4 replications, a total of 16 plots. The treatment plots are of rectangular shape of each being 0.045 ha (30x15 m) in area and enclose assessment plots of 0.02 ha (20x10 m), Fig.1 . There are 3 m buffer zones which separate the treatment plots.

In April 1970 the plots were thinned (low thinning but no more details available from the Forestry Commission) and fertilized. Fertilizers were applied in the following proportions and quantitites:

Treatment 1: P- phosphate, as 375 kg/ha of unground rock phosphate, supplying 50 kg P/ha

Treatment 2: K- potassium, as 200 kg/ha of muriate of potash, supplying 100 kg K/ha

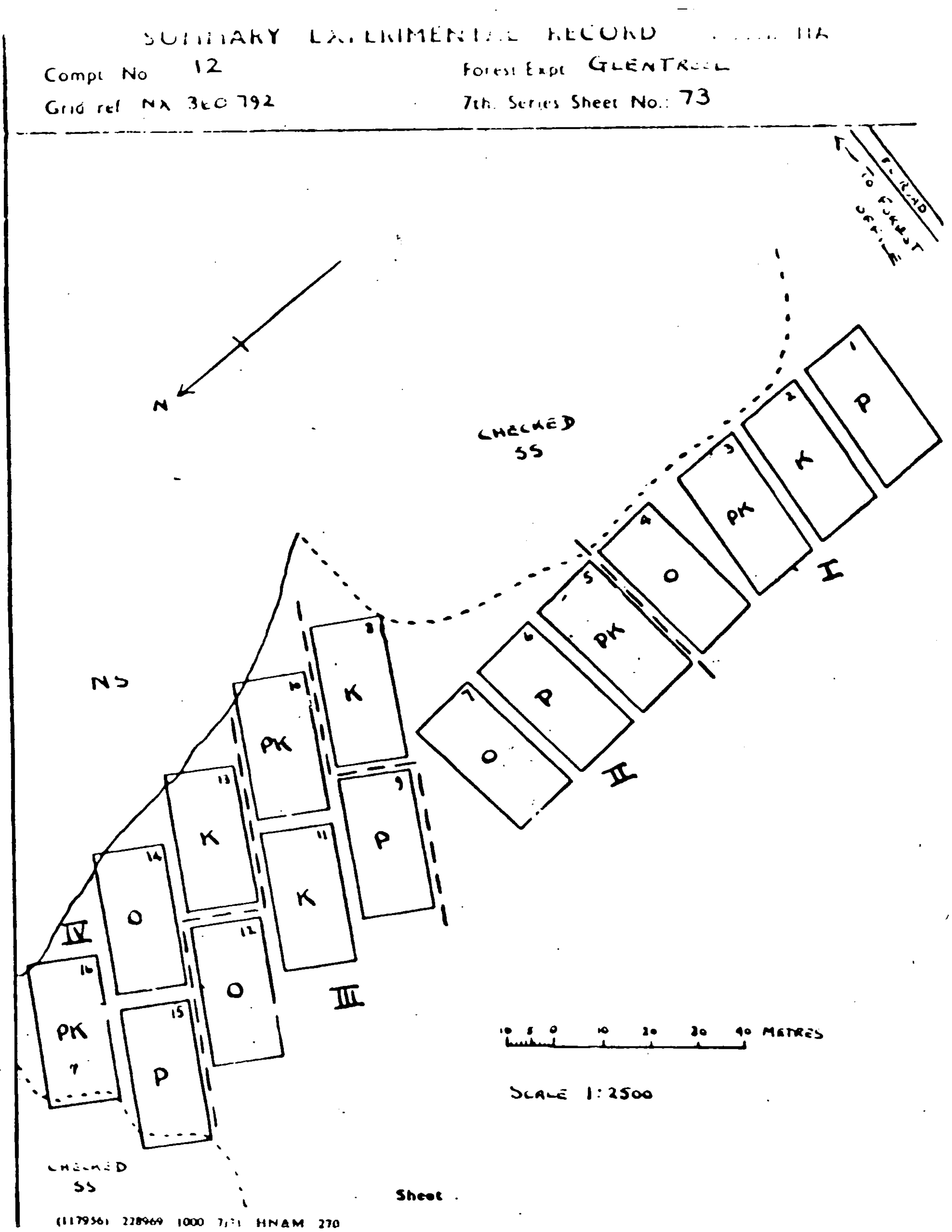

Fig. 1 Layout of the experiment

Treatment 3: PK- combination of P and K treatments supplying 50 kg P/ha and 100 kg K/ha

Treatment 4: 0- control (no top dressing)

In this study the effects of combined fertilization (PK) have been examined and in the following blocks and plots (Table 2)

TABLE 2

PLOTS EXAMINED IN THE STUDY

BLOCK 1	BLOCK 2	BLOCK 3	BLOCK 4
Plot 3 (PK)	Plot 5 (PK)	Plot 10 (PK)	Plot 16 (PK)
Plot 4 (0)	Plot 7 (0)	Plot 12 (0)	Plot 14 (0)

All the trees in the above plots were numbered and a cross mark on each tree showed the BH position, allowing repeated measurements of DBH to be taken at the same position.

In 1970 the trees in these plots were assessed as General Yield Class 18 (Forestry Commission data). In November 1975 all plots, buffer zones and surround areas, were thinned again. According to the Forestry Commission a BA between 0.209 and 0.223 m^2 had been removed per treatment (1,2,3,4).

2.2 TREE CLASSIFICATION

All the trees in the plots under examination were classified for dominance in the following 4 classes:

Class 1: Dominant trees

These were the tallest and most vigorous trees in crop and usually had a large proportion of their crown free

Class 2: Co-dominant trees

These were trees in the upper canopy which they helped to complete, but they were below the crown level of the dominants.

Class 3: Sub-dominant trees

These trees were not in the upper canopy but their leaders still had access to light which had not filtered through the foliage of adjacent trees

Class 4: Suppressed trees

These were trees whose leaders had no direct access to light and stand beneath the crowns of adjacent trees

The dominance classification of a tree was of course relative to those trees immediately surrounding it. The same criteria for classification were applied to all plots irrespective of treatment. The DBH frequency distributions of the trees' for the years 1971 & 75 are given in Appendix 2.

2.3 SAMPLING PROCEDURE

According to the Forestry Commission's sampling scheme for this study, ten trees per control plot and ten trees per fertilized plot were sampled. These trees ought to cover the DBH frequency distribution in each plot and consequently to represent the tree classes described in the previous section. It was also decided that three of the ten trees to be representative of the suppressed and subdominant classes (taken as one class) and the rest to represent the co-dominant and dominant classes. Since the trees of each plot had been numbered it was easy to sample randomly, using the numbers of the trees, under the above restriction.

In December 1978, after the end of the growth period, a total of 80 trees (40 control and 40 fertilized), were selected in the forest following the above described sampling procedure and felled. From the 40 control trees 14 or 35% were dominants, 14 or 35% were co-dominants and 12 or 30% suppressed-sub dominants, while the corresponding proportions for the fertilized were 17 or 42.5%, 11 or 27.5% and 12 or 30%.

2.4 DISC COLLECTION

Since one objective of this study was the examination of the internal pattern of increment it was necessary to take measurements of diameter and cross sectional area from discs taken so as to examine the distribution of increment both, longitudinally and radially throughout the tree. Thus any changes in stem form caused by a redistribution of the increment as a result of fertilization would be detectable. Also the diameters to be measured on these discs ought to be close enough in order to permit adequate interpolation for graphic representation of "stem profiles", for comparative purposes, as well as for fitting of taper equations. The same diameter measurements ought to be "representative" in the sense that they should be taken at positions on the tree avoiding nodes or other possible irregularities of the stem (Reukema, 1971).

To meet these requirements it was decided to select one disc from each mid-internodal position of the stem of each of the trees. Thus a number of discs of about 3 cm thickness was cut from each tree as follows.

The first disc was cut at a standard distance of 0.65 m above the ground level (half the BH distance) and considered to account for the butt-swell, the second was cut at the BH position and thereafter one disc from each mid-internodal position to a height where the diameter became less than 7 cm. Every disc was marked on the top with its number and the number of tree and treatment. As soon as they were cut, the discs were put into plastic bags, each of them labelled with the block, plot, treatment and tree numbers. The bags were sealed to prevent drying out.

CHAPTER 3

METHODS OF DATA COLLECTION

3.1 INTRODUCTION

In order to fulfil the objectives of this study the methods to be applied should satisfy the following conditions:

a. The measurements of volume variables, such as DBH and height as well as their increment, should be as accurate as possible and combined with the use of the proper volumetric formulae (Chapter 4, para 5) for a sound estimation of tree volume.

b. The examination of the increment of trees and its distribution should not be limited at one particular position of the tree stem, such as the DBH position, but should cover the total length of the stem (Assmann,1970, Fayle and Mcdonald, 1977). The internal pattern of increment should be revealed, since increment pattern and stem form of the trees closely related, Assmann (1970) who very clearly states that:

"In order to obtain an insight into the growth of the trees we must first analyze the pattern of growth and the resulting shape of individual trees".

All these above called for analysis of the increment pattern using the "records" which are accumulated during the life of every tree:Incremental rings and application of stem analysis methods.

3.2 DEFINITIONS

Throughout this study the following definitions are applied:

1. <u>Total height</u>

The distance between the height at which the tree was cut and the top of the tree, in metres.

2. Crown depth

   The distance between the top of the tree and midway between the lowest live whorl and lowest live branch, measured in metres.

3. Lowest live branch

   A branch retaining some amount of live foliage.

4. Lowest live whorl

   The lowest whorl with all branches live.

5. Whorl height

   The distance between a whorl and the height where the tree was cut (lower end of the bole), measured in metres.

6. Disc height

   The distance between a disc and the height where the tree was cut (lower end of the bole), measured in metres.

3.3 FIELD MEASUREMENTS

The following measurements were taken in the field:

1. The height of the position where each disc was to be cut.
2. The whorl height from the first distinguishable whorl to the last.
3. The diameter at the BH position (overbark).
4. The total height of the tree.
5. The crown depth.

All these measurements were taken as follows:

Before felling the DBH was measured with a tape. After felling the position of the lowest live whorl as well as the position of the lowest live branch were located and the mid point between them was marked on the tree. Thus the lower end of the crown depth was located. The big branches were removed in order to facilitate placing the tape along the stem of the tree, and the crown depth was measured and recorded, as well as the height of each whorl. Finally the total height of the tree was measured and recorded. After taking all these measurements the discs were cut from the predetermined positions.

3.4 ANNUAL RING MEASUREMENTS

It was decided to measure all the annual ring widths on four radii, in the direction of the longest axis of the section and the axis at a right angle to the longest axis. This gives an accurate estimation of diameter or cross sectional area and their increments (Siostrzonek, 1958, Reukema, 1971) for the volume and volume increment estimation of the current or any previous year of the tree's life. The measurements were always taken on the upper side of the disc and from bark to pith. An example of recording the ring width is given in Fig. 1, Appendix 1. For these measurements a special measuring device in conjunction with a binocular microscope has been used (plate 1).

The measurements were recorded to the nearest 0.01 mm, though a test of the instrument using to repeated measurement, on one radius showed that the SD of each measurment was approximately 0.03 mm, irrespective of measurement value. A plot of the SD against the mean values (Fig. 2, Appendix 1) did not show any particular trend.

3.5 BARK MEASUREMENTS

Volume estimation of trees is usually based on overbark diameter measurements taken at several positions on the stem. Because of the lack of previous knowledge of the effect of fertilization on the bark of Sitka spruce and since estimation of bark thickness is considered to be critical in diameter and volume increment studies (Reukema, 1971)., it was decided to measure the bark thickness on each of the four radii of each disc (Whyte and Mead, 1977). These data could be used for the estimation of the average bark thickness at each height and finally

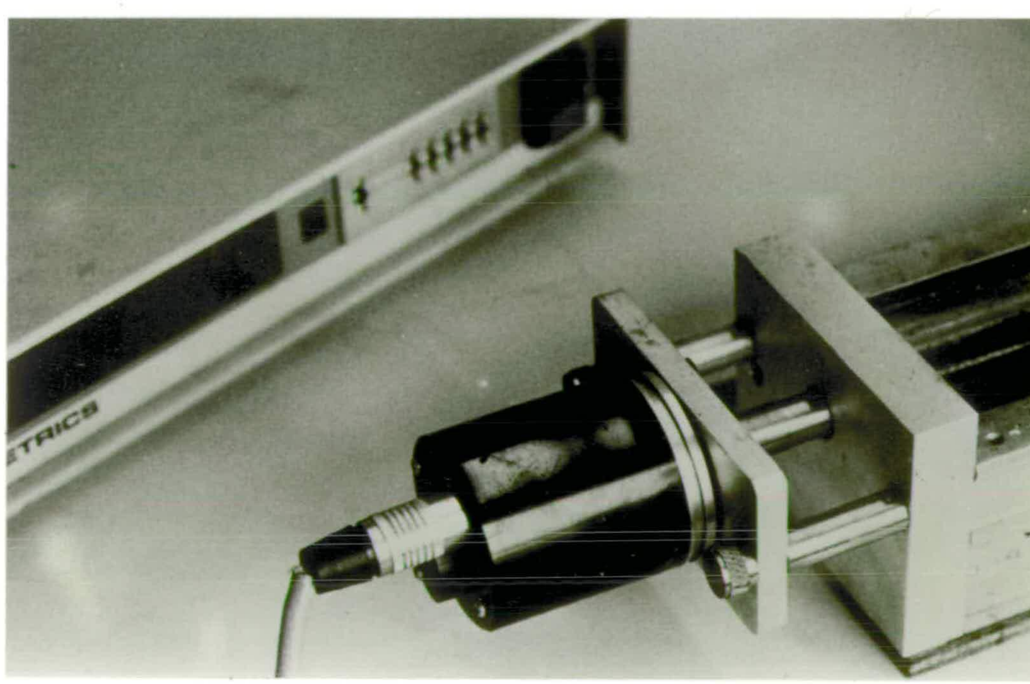

Plate 1. Measuring device used for ring width measurements

for the estimation of the total bark volume of the trees.

3.6 DATA HANDLING

The data derived from ring width measurements of discs and their height were punched on cards and stored in computer files in the format shown in Fig. 2. In the data file, part of which is presented in Fig.3, the format is as follows:

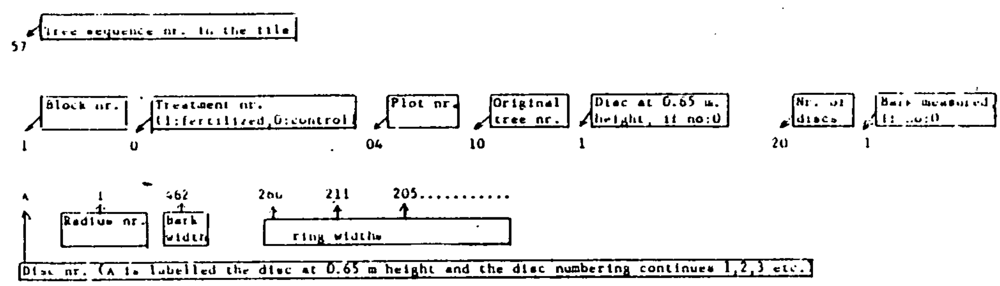

Fig. 2. Data format

These data were grouped by blocks of the Glentrool experiment and by treatment. Within each treatment the ten trees (sample) were ranked by their DBH in 1978.

With the aid of a number of subroutines it was possible to calculate mean radius, diameter and cross sectional area for any year. With further infcrmation on the height of each disc tree volumes were calculated and the pattern of growth at different stages reconstructed.

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*****************************************************************************
53
3 1 10 13 1 20 1
A 1 421 083 085 132 166 223 263 227 283 168 197 170 167 225 251 227 123 268 206
    388 533 477 417 320 402 534 515 416 237
A 2 325 036 039 046 130 146 131 121 109 068 105 112 099 171 154 158 128 230 206
    240 343 315 275 354 443 503 485 345 321
A 3 402 038 049 037 177 121 123 127 123 091 093 131 135 139 166 224 162 181 203
    296 412 376 295 251 328 500 505 363 309
A 4 370 077 090 093 152 203 144 171 211 101 135 157 146 199 136 198 135 217 234
    328 497 595 461 367 403 513 512 399 316
1 1 394 062 093 036 176 205 203 149 230 109 091 135 163 205 169 180 165 300 276
    305 498 435 512 531 512 577 549 210
1 2 578 032 047 043 094 117 133 120 133 035 080 078 134 150 172 173 129 163 164
    211 789 534 294 435 632 616 643 217
1 3 439 086 091 124 197 275 224 206 132 114 115 133 139 241 176 206 166 216 200
    285 509 565 620 509 529 522 472 161
1 4 410 024 044 036 084 088 092 127 103 071 087 113 144 139 142 173 128 151 208
    317 382 330 342 449 492 514 542 121
2 1 524 065 076 123 177 200 222 170 211 138 136 174 197 156 195 225 191 294 308
    404 573 562 671 273 679 423 313
2 2 424 051 053 030 082 091 109 111 121 070 095 107 122 169 156 136 143 139 241
    375 509 692 567 491 521 543 316
2 3 333 045 049 350 036 124 155 167 265 126 120 147 139 176 171 187 172 211 236
    334 505 553 521 365 393 414 327
2 4 305 041 053 153 136 162 166 154 174 098 103 158 149 214 202 201 164 245 293
    387 543 492 552 436 606 516 272
3 1 325 064 073 094 147 166 161 159 192 167 135 150 205 226 205 165 192 274 292
    406 660 597 671 403 444 307
3 2 440 024 020 052 121 194 136 143 171 073 111 139 124 163 167 163 184 235 252
    363 536 618 517 292 339 417
3 3 333 039 057 056 082 114 123 126 120 094 139 124 135 163 154 202 183 202 260
    406 604 555 512 323 367 400
3 4 325 038 061 055 123 148 191 146 143 119 124 144 167 208 233 187 206 245 295
    422 621 627 709 393 357 37
4 1 307 027 052 058 132 206 211 133 223 154 148 148 156 234 246 214 228 272 298
    432 590 474 361 240 572
4 2 298 029 043 036 071 094 134 149 126 333 091 124 146 109 215 183 263 315 263
    549 630 555 470 334 324
4 3 308 041 040 055 103 113 114 134 134 094 103 095 139 157 167 209 240 249 260
    439 640 552 423 293 341
4 4 201 031 040 059 109 122 163 188 189 130 136 158 167 256 279 276 355 338 347
    535 683 503 436 275 512
5 1 334 028 055 051 102 124 187 211 233 147 148 163 174 247 235 239 330 319 321
    588 629 441 274 298
5 2 327 026 041 075 133 133 176 155 160 039 102 114 166 221 236 230 357 403 370
    598 687 449 222 367
5 3 431 036 058 074 121 146 156 150 182 103 152 142 161 258 226 280 422 301 263
    605 597 421 253 291
5 4 390 023 034 043 095 109 137 173 178 093 102 133 167 251 228 232 277 295 438
    622 709 459 342 379
6 1 331 051 089 085 127 139 162 182 187 113 139 193 108 258 211 303 446 382 357
    567 543 358 347
6 2 355 028 046 062 122 163 155 179 228 111 136 154 177 264 239 242 365 335 451
    662 595 379 249
6 3 336 034 063 092 113 172 162 176 195 097 127 134 129 269 274 279 375 447 366
    590 634 359 328
6 4 298 040 033 033 096 139 131 186 191 144 149 137 168 246 273 302 365 471 418
    435 574 447 315
7 1 336 023 050 159 138 161 177 187 197 141 121 151 177 351 334 387 452 568 492
    537 494 410
7 2 345 020 020 041 100 132 144 167 159 134 152 121 164 243 293 308 422 505 394
    534 499 279
7 3 300 027 042 047 113 172 243 194 241 147 147 144 172 281 317 295 414 453 413
    531 494 412
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Fig. 3. Part of data file of ring width measurements

CHAPTER 4

SINGLE TREES - VOLUME ESTIMATION

4.1 INTRODUCTION

One of the basic requirements in growth and yield studies in forestry is to estimate the volume of a forest stand. This can be done either by complete enumeration or by sampling. Complete enumeration is used in the case of a small forest, or when we are dealing with a small number of trees, when it is then possible to estimate the stand volume as the sum of the volumes of the individual trees. When dealing with a larger number of trees, characteristics of the stand are estimated by sampling. The main problem in either case is to estimate the volume of single trees. After a solution is found to this problem, the rest depends on sampling error and sampling intensity, to achieve the desired standards of precision.

Methods for estimating volume involve taking and analysing a large number of measurements. The use of computers has simplified the job of doing the calculations. The taking of measurements has been simplified by the use of optical instruments, such as the Barr and Stroud dendrometer and the relascope. The use of such instruments reduces the need to fell trees for estimating volume. However, it may still be necessary to fell trees when a high degree of accuracy is required, such as for specific research objectives (e.g. growth studies using stem analysis or the preparation of volume tables), or when there are doubts concerning the accuracy of existing volume tables (following fertilization for example).

Spurr (1952) stated that few subjects in forest mensuration have received as much attention as the estimation of tree volume and according to him,

"the reason is evident, because of the consideration of the tree as a complex and highly variable geometric solid. Thus the assessment of its volume in terms of very few measurements and by simple algebraic techniques is by its nature difficult if not impossible".

It is very common in forestry practice to adopt the idea of similarity of the tree stem with a solid of revolution bounded by a curve (the "diameter: height curve"). If a tree stem is bisected by a vertical plane which passes through the pith, the section is bounded by a curve which theoretically is symmetrical with respect to the vertical axis (Fig. 4).

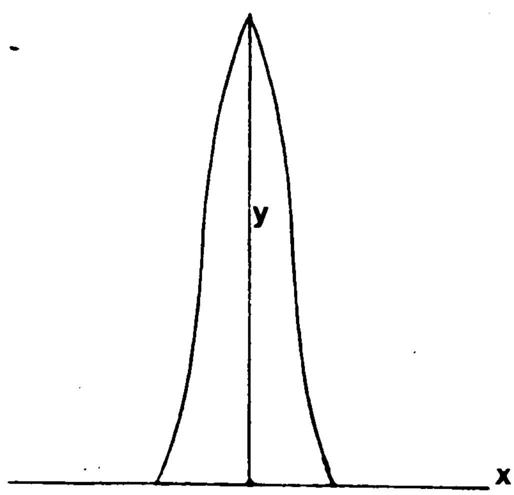


Fig. 4. Longitudinal section of a stem as solid of revolution

If this curve revolves around the vertical axis (Y) a solid of revolution results.

Coniferous trees tend to have a definite central stem which probably does approximate quite closely to a geometric solid of revolution, their stem form is termed "excurrent", (Husch, 1963). Broadleaved trees, on the other hand, are "deliquescent" (Husch, 1963), having a different branching habit and less definite central stem. The following discussion is restricted to trees having an excurrent stem form.

Considering the tree as a solid of revolution means that its volume can be calculated if the nature of the curve relating radius (diameter) to height is known or assumed. Three approaches have been used:

a. Theoretical development of the diameter: height equation from a consideration of the biological or structural roles of the stem.

b. Use of empirical equations for calculating volume for the whole tree.

c. Use of empirical equations to describe the diameter: height relationship of sections of the tree, the volume of the whole tree being found by summing the volumes of the sections.

4.2 THEORETICAL APPROACH TO THE DIAMETER: HEIGHT RELATIONSHIP

Several theories have been developed concerning the development of the tree stem. Each of these theories is based on different factors which influence stem development, resulting in different relationships between diameter and height. Some of these theories are described in the following sections.

Metzger's theory

Metzger (1893) considered the effect of the wind pressure as the main factor for the development of stem. According to his theory the stem was considered as a carrier of equal resistance to bending, and the force of the wind responsible for the bending action was assumed to be greatest at the center of gravity or the midpoint of the crown. The stem functioned as a lever arm and the stress created by the wind on the crown was propagated downwards culminating in a maximum at the stem base. The strengthening of the stem in the form of increased diameter growth, paralleled the stress gradient downward in the stem. Plotting the cube of diameter (d^3) of sample trees, Metzger demonstrated that the d^3 values for various heights in the branch free bole corresponds to a straight line, therefore conforming to his hyphothetical beam of uniform resistance to bending. Above the point where maximum wind force was presume to be applied (centre of gravity of the tree or Focal point), wind pressure was less and thus stem diameter decreased, while below that point the stem should conform with the following relationship:

$$h = ad^3 \qquad (2)$$

where: h = distance from the centre of gravity

d = diameter at h

a = coefficient expressing the taper

Hohenadl's theory

Hohenadl (1922), considered the weight of the stem and crown was the decisive factor for the stem form development. It was assumed by Hohenadl (Larson, 1963) that in the beginning of each seasonal growth period the requirements for conduction tissue (earlywood) were predominating and the requirements for strength tissue (latewood) were absent or slight. As growth proceeded in the crown and stem the weight would increase accordingly and greatly intensify the compressive forces. The latter would necessitate reinforcement of the strength tissue in the stem, and the tree would satisfy the demand not only for additional growth in the areas where the compressive forces were greatest but also by a more uniform distribution of conduction and strength tissues. At the end of the growing season, after leaf and twig fall, stem and crown weight would decline and may again be negative with regard to compressive strength which means there would be no demand for additional strength tissue. Therefore, when certain growth conditions resulted in a decrease in the specific weight and compression strength of the wood, a concomitant decrease in the stem cross section could also be anticipated.

Summarizing it could be said that Hohenadl regarded the stem as a beam of uniform resistance to compression rather than to bending due to forces exerted by its crown weight. Finally Hohenadl gave a logarithmic formula for the diameter estimation at different heights.

Gray's theory

Gray (1956) agreed with Metzger's theory as far as wind agreed was concerned, as the main factor in stem form development and summarized his conclusions as follows (Larson, 1963):

> "Critical reflection suggests that Metzger's deduction is an artificial one, because only if a tree were embeded in material sufficiently strong to ensure that the attachment at the base would hold against forces greater than necessary to beak the stem, would it require the dimensions of a cubic paraboloid to offer uniform resistance to lateral pressures centred on the crown. As a tree is embedded in a relatively weak material it would appear that Metzger's stem is unnecessarily strong, and so it does not represent the most efficient structural member from a mechanical point of view".

Finally Gray (1956) states abundant evidence indicating that the dimensions of the branch free bole conform to a quadratic paraboloid, and gives the following expression for the diameter:height relationship

$$h = ad^2 \quad (3)$$

where: h = distance from "parabolic height" (which is the distance from the intersection between the vertical axis and the d^2:height curve)

d = the diameter at distance h from parabolic height

a = cofficient expressing wind pressures on crown

As it appears from the above theories, having arrived at a diameter: height relationship the volume of the tree can be estimated by integration. Complete agreement cannot be expected between theoretically determined and the actual diameters, since the tree stem is a component of a living organism and its shape does not result from the influence of mechanical forces alone. Physiological processes and variations in external influences affecting the growth of the tree are also important. Thus the actual shape of the tree is more complicated than for example, a beam of uniform resistance (Metzger's theory) and as a result the volume estimation of it becomes more difficult.

4.3 EMPIRICAL WHOLE TREE FORMULAE

Numerous attempts have been made to use formulae for tree stem volume estimation adopting a small set of height and diameter measurements. Each formula calculates the volume of the stem but unlike the previous (a) approach it is not based on any idea of what the tree should be like, but rather assumes that the tree stem has a simple geometric shape. Based on this latter principle a number of formulae which apply to the volume estimation of stereometric solids were adopted and used in forestry practive foi the volume estimation of the tree ste. Examples of this approach are:

- Newton's formula

$$V = \frac{g_o + g_1 + 4g_{1/2}}{6} \times H \qquad (4)$$

where V = the stem volume

$g_{1/2}$ = the sectional area at the middle of the stem

g_o = the sectional area at the base of the stem

g_1 = the sectional area at the top of the stem

H = total height above stamp

This formula, which is actually the formula for the volume estimation of a prismoid, can be used to estimate the volume of any of the forms which a tree stem assumes (Husch, 1963).

With the idea of modifying standard formulae (applied to stereometric solids), to minimize the work needed for obtaining the required measurements for an accurate stem volume, the following formulae have been developed:

- Pressler's formula

$$V = \frac{2}{3} \times g_o \times h_{dl/2} \qquad (5)$$

where V = stem volume

g_o = sectional area at the base of the stem

$h_{dl/2}$ = height above ground at point where diameter is 1/2 of BH diameter

- Gossfeld's formula

$$V = \frac{H}{4} (3g_{1/3}+g_o) \qquad (6)$$

where V = stem volume

H = total height

$g_{1/3}$ = sectional area at 1/3 of total height from base, and

g_o = sectional area at base of stem

The above formulae are applied for any form that the tree stem assumes.

This empirical approach is likely to be even less satisfactory than the theoretical approach. Not only does it ignores the varying influences that the tree is subject to during its growth but it neglects the mechanical influences that are likely to have been important in the evolution of stem form. It does, however, provide a quick though approximate method for estimating volume.

4.4 EMPIRICAL RELATIONSHIPS (FORMULAE) APPLIED TO STEM SECTIONS

An alternative to the whole stem approach is to consider the stem as consisting of a number of sections. The extreme of this approach is of course to regard the stem as consisting of a large number of very thin cylindrical discs. The volume of the tree is then the sum of the cross sectional area of each disc multiplied by its height. This extreme approach is undesirable for two reasons:

1. The amount of effort required to take all the diameter measurements, and
2. Its lack of insight into general features of stem shape

In practice, it may be more convenient to treat the stem curve as a composite of several curves. It is usually concave in the lower part, convex for most of the rest of the tree stem, and approaches a straight line near the top of the stem. All these parts of the stem curve can be described by the following general equation

$$X^a = KY^b \qquad (7)$$

where X = the cross section radius of the stem

Y = the distance from the top of the curve and finally K is a coefficient.

This equation describes a family of curves known as parabolas (Fig. 5)

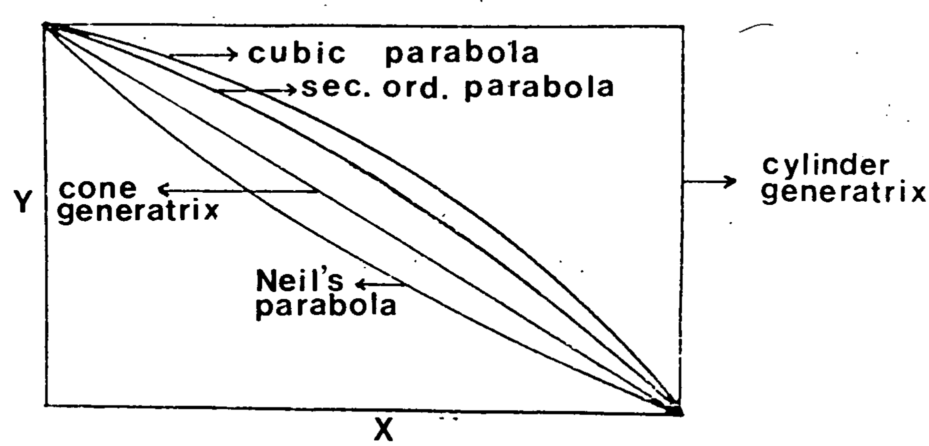


Fig. 5. Family of curves produced by the equation $X^a = KY^b$

According to the changes in the exponents a and b of this equation it is possible to construct different curves closely approximating parts of the actual stem curve. Finally revolution of these curves around the

-Y-axis produces different solids approximating parts of tree stem, such as those of Fig. 6.

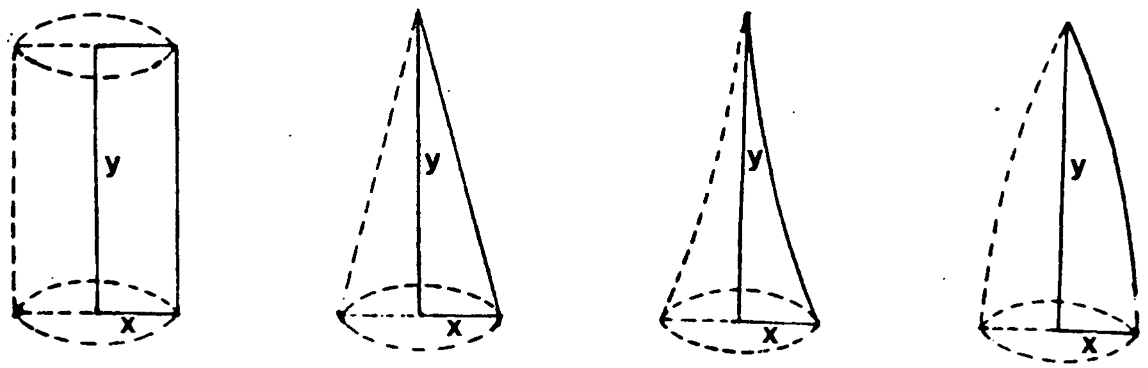


Fig. 6. Solids of revolution approximating parts of tree stem

The most common and widely used formulae for sectional volume estimation have been based upon this theory. Examples of such formulae are:

- Smalian's formula

$$V = \frac{g_b + g_t}{2} \times h \quad (8) \text{ and}$$

- Huber's formula

$$V = g_m \times h \qquad (9)$$

where V = the volume of the section of the stem

g_m = the cross sectional area at the middle of the section

g_b = the cross sectional area at the base of the section

g_t = the cross sectional area at the top of the section

Both formulae estimate the volume of a frustum of a paraboloid (Frustum is the part of solid left if its top is taken off e.g. by a cut parallel to base), and were adopted in forestry for the sectional volume estimation of tree stems (Spurr , 1952, Husch , 1963).

Since the final estimation of the stem volume is the result of adding up the volumes of different sections both (8) and (9) give good results provided that stem sections are frustrums of paraboloids. The results of (8) or (9) will be incorrect if the stem sections are conic or neiloidic frustrums. Smalian's formula based on end cross sectional areas will overestimate volume, while Huber's formula which uses the middle cross sectional area will underestimate. Finally Newton's formula (4) can be also used for sectional volume estimation and in this case accuracy will be improved since this formula makes use of three sectional areas for the section's volume estimation, but the number of measurements will increase.

Husch (1963) suggested that the volume of the tree stem will be better approximated with the use of the sectional volume estimation, but, recognising the stem shape as it appears in Fig. 7.

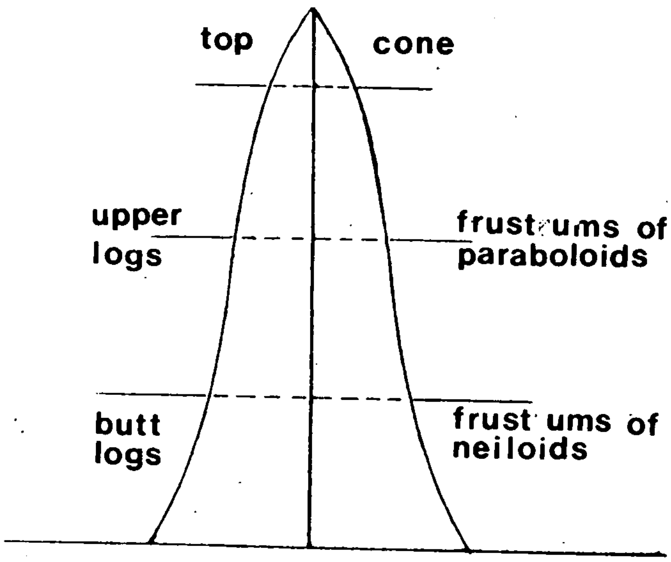


Fig. 7 , Geometric forms assumed by portions of a tree stem (Husch , 1963) and applying the proper formula for the corresponding part (section) of the stem.

After the examination of the foregoing main empirical relationships which areapplied for the volume estimation of the stem it could be said that the major difficulties arise from the weakness of a general formula for the volume estimation of a highly variable tree stem, or parts of it. Since this study deals with a "sensitive" situation as far as the volume estimation of the single trees was concerned, a decision was taken to examine, investigate and develop an appropriate method of volume estimation.

Adoption of any theory considering the development of a tree stem as approaching a paraboloid (Metzger's theory-Gray's theory), might lead to serious errors and perhaps mask probable increases caused by fertilization. Application of Smalian's or Huber's formula for sectional volume estimation made it necessary that tree stem or parts of it resembled a paraboloid - for correct volume estimation. Newton's formula was rather cumbersome without defining the section's length for an adequate degree of accuracy and without assurance for "representative" points [illegible] i.e. points along the stem at which diameters are not affected by any kind of irregularities of the stem. Husch's suggestion concerned with the recognition of the stem as in Fig.7 did not define where the stem shape turns from neiloid to paraboloid or to conoid in order to apply the corresponding formula for that particular section of the tree stem.

All these lead to examination for a method which might cope with the complexity of the stem shape under the conditions of the particular experiment of fertilization.

4.5 A PROPOSED METHOD FOR THE SECTIONAL VOLUME ESTIMATION OF THE TREES

Introduction

In this study the volume estimation method to be applied should be capable of:

1. Coping with the complexity of the tree stem (as already discussed in the previous section) and,

2. accounting for possible stem form changes caused by the combined effect of fertilization and thinning, since possibly either of these two treatments might affect it.

Several researchers in the past stressed this complexity in stem form below and within the crown as well as in the butt region of the ste, (Petrini 1921, Behre 1923, Prodan 1944, Larson 1963). Since stem form is the result of the internal increment pattern of the tree, factors which might affect the internal pattern of increment might also affect and contribute to this complexity. It has already been reported that there is a redistribution of increment on the tree stem caused by thinning (Hartig 1901, Hagberg 1942, Naslünd 1943 , Assmann 1970). Miller and Cooper (1973) reported changes in the taper of Corsican pine after the application of fertilizers and Whyte and Mead (1977) reported changes in the stem form of Radiata pine following fertilization. Changes in stem form or taper of Sitka spruce do not appear to have been reported so far.

From all the above it can be said that any silvicultural treatment that might alter the prevailing conditions in the crown might be reflected by a concomitant change in the stem form of the trees. It is likely that fertilization might improve the growth rate by increasing the photosynthetic efficiency of the foliage and/or by increasing the leaf area and therefore probably affecting the stem form. Brix (1976) presents evidence of such changes taking place in thinning-fertilization experiment of 24 year-old Douglas fir. It is also possible that fertilization may "advance" the stage reached by trees in relation to those in the control, a stage which the latter would reach at some later point in time. Hence it might be more appropriate to consider that form is not actually altered by fertilization, only moved along its normal progression at a faster rate (Miller , 1981).

The method

It is well documented from previous research that stem form changes from tree to tree under the influence of crown size or tree class (Tiren 1928, Burger 1951, Horn 1961), and from forest to forest, as it is influenced by differences in site, structure and management (Badoux 1935, Zimmerle 1951, Calaham and Liddicoet 1961), and within the same forest from one stage of age to the next (Jonson 1927, Leibundgut 1945, Bickerstaff 1946, Fanta 1958). Also there are no standard positions on the stem where the stem curve turns from neiloid into paraboloid and/or to conoid.

Grosenbaugh (1966) summarizing his work on tree volume estimation says that height-diameter measurements taken for volume estimation have to be enough to locate all points of inflection. He also comments that:

> "concavity or convexity becomes relatively unimportant in sections short enough so that the smaller diameter is at least 8/10 of the larger diameter".

It was felt that for an accurate volume estimation of a section of a stem - assuming that the stem is considered as an aggregate of several geometrical solids of revolution - the discrepancies caused from using different volume formulae for the volume estimation of a section had to be minimized. For this reason an alternative to using a particular volume formula corresponding to the section of the tree resembling a particular solid of revolution was investigated. This involves measuring diameter up the tree at points of equal reduction in diameter (called the diameter step). If one used an infinitely small diameter step, which in effect means taking an infinite number of diameters,it would be possible to have a highly accurate volume estimate irrespective of the formula used to calculate the volume of each stem sections. Alternatively with a very coarse diameter step the different formulae applied to a given stem section may produce quite different estimates of volume for the section.

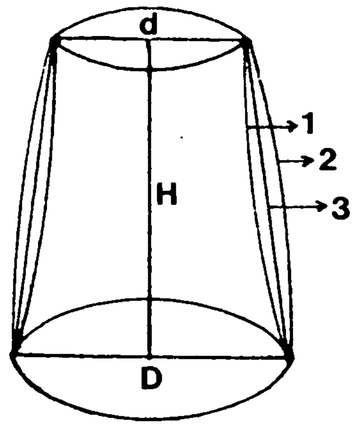
In between those two extremes there will be a diameter step which represents the prefered trade-off between effort (the number of diameters to be measured) on one hand and the discrepancy (in % terms) between the extreme volume formulae on the other hand. (Here by extreme formulae is meant the formulae for neiloid and paraboloid). It was therefore decided to test different volume formulae for estimating the volume of sections based on diameter steps rather than adopting standard lengths (e.g. tenths of total height), or any other predefined length. The volume formulae tested (Forest Mensuration Handbook, Hamilton G.J. 1975) are detailed below and their application identified in the frustrums of Fig.8 :

1. For the frustrum of Neiloid, $V=\frac{\Pi}{16} \times H \times (D^2+d^2+2dD)$ (10)

2. For the frustrum of Paraboloid, $V=\frac{\Pi}{8} \times H \times (D^2+d^2)$ (11)

3. For the frustrum of Conoid, $V=\frac{\Pi}{12} \times H \times (D^2+d^2+Dd)$ (12)

where D = the large-end diameter of the frustrum



d = the small-end diameter of the frustrum

H = the height of the frustrum

Fig. 8. Frustrums for which volume formulae were tested

A program for volume estimation was tested by computer using the above mentioned three volume formulae for different heights and diameter steps. From the part results presented in Tables 3 and 4, it can be seen that when the difference between the two end diameters (diameter step), is 2.5 cm, the percent difference of volume between the three frustrums tends to be less than 1%.

The next step was to investigate the relationship between effort (the number of measurements) and accuracy. From the results presented on Table 5, Fig.9 was constructed. This Figure shows the relationship between percent volume discrepancy (neiloid/paraboloid) and the ratio of diameter step/big diameter. With the help of data extracted from this Figure, the next Figure (Fig.10) was constructed. This final Figure provides the diameter step that should be used for a given DBH and for a required degree of agreement between the extreme formulae (paraboloid and neiloid).

For example if one required a maximum discrepancy of 1% (i.e. the volume estimated by the two formulae agreed to within 1%) and the stem has a diameter of 25 cm at the BH position, then a diameter

TABLE 3

VOLUME ESTIMATION USING FORMULAE FOR CONOID, PARABOLOID AND NEILOID FRUSTRUMS

1	2	3	4	5	6	7	8	9	10
Dcm	dcm	Hm	VOLUME OF CONOID m^3	VOLUME OF PARABOLOID m^3	VOLUME OF NEILOID m^3	VOLUME DIFFERENCE m^3		VOLUME DIFFERENCE %	
						PARABOLOID-CONOID	NEILOID-CONOID	PARABOLOID-CONOID	NEILOID-CONOID
12.5	10	1	0.0099	0.0100	0.0099	0.0001	-0.0000	0.82	-0.41
15.0	10	1	0.0124	0.0127	0.01[illegible]	0.0003	-0.0002	2.60	-1.32
17.5	10	1	0.0152	0.0159	0.0148	0.0007	-0.0004	4.72	-2.45
20.0	10	1	0.0183	0.0196	0.0176	0.0013	-0.0006	6.90	-3.54
22.5	10	1	0.0217	0.0238	0.0207	0.0020	-0.0010	8.98	-4.81
25.0	10	1	0.0255	0.0284	0.0240	0.0029	-0.0015	10.91	-5.94
27.5	10	1	0.0296	0.0336	0.0275	0.0040	-0.0020	12.68	-7.01
30.0	10	1	0.0340	0.0523	0.0314	0.0052	-0.0026	14.29	-8.00
12.5	10	2	0.0199	0.0201	0.0198	0.0002	-0.0001	0.82	-0.41
15.0	10	2	0.0248	0.0255	0.0245	0.0006	-0.0003	2.60	-1.32
17.5	10	2	0.0304	0.0319	0.0296	0.0015	-0.0007	4.72	-2.45
20.0	10	2	0.0366	0.0392	0.0353	0.0026	-0.0013	6.90	-3.54
22.5	10	2	0.0435	0.0476	0.0414	0.0041	-0.0020	8.98	-4.81
25.0	10	2	0.0510	0.0569	0.0480	0.0059	-0.0029	10.91	-5.94
27.5	10	2	0.0592	0.0672	0.0551	0.0080	-0.0040	12.68	-7.01
30.0	10	2	0.0680	0.0785	0.0628	0.0105	-0.0052	14.29	-8.00
12.5	10	3	0.0299	0.0302	0.0298	0.0002	-0.0001	0.82	-0.41
15.0	10	3	0.0373	0.0383	0.0368	0.0010	-0.0005	2.60	-1.32
17.5	10	3	0.0456	0.0478	0.0445	0.0022	-0.0011	4.72	-2.45
20.0	10	3	0.0549	0.0589	0.0530	0.0039	-0.0020	6.90	-3.54
22.5	10	3	0.0652	0.0714	0.0622	0.0061	-0.0031	8.98	-4.81
25.0	10	3	0.0765	0.0854	0.0721	0.0088	-0.0044	10.91	-5.94
27.5	10	3	0.0888	0.1008	0.0828	0.0120	-0.0060	12.68	-7.01
30.0	10	3	0.1020	0.1177	0.0942	0.0157	-0.0078	14.29	-8.00
12.5	10	5	0.0499	0.0503	0.0497	0.0004	-0.0002	0.82	-0.41
15.0	10	5	0.0621	0.0638	0.0613	0.0016	-0.0008	2.60	-1.32
17.5	10	5	0.0760	0.0797	0.0742	0.0037	-0.0018	4.72	-2.45
20.0	10	5	0.0916	0.0981	0.0883	0.0065	-0.0033	6.90	-3.64
22.5	10	5	0.1087	0.1190	0.1036	0.0102	-0.0051	8.98	-4.81
25.0	10	5	0.1275	0.1423	0.1202	0.0147	-0.0073	10.91	-5.94
27.5	10	5	0.1480	0.1680	0.1380	0.0200	-0.0100	12.68	-7.00
30.0	10	5	0.1701	0.1962	0.1570	0.0262	-0.0131	14.29	-8.00

TABLE 4

VOLUME ESTIMATION USING FORMULAE FOR CONOID, PARABOLOID AND NEILOID FRUSTRUMS

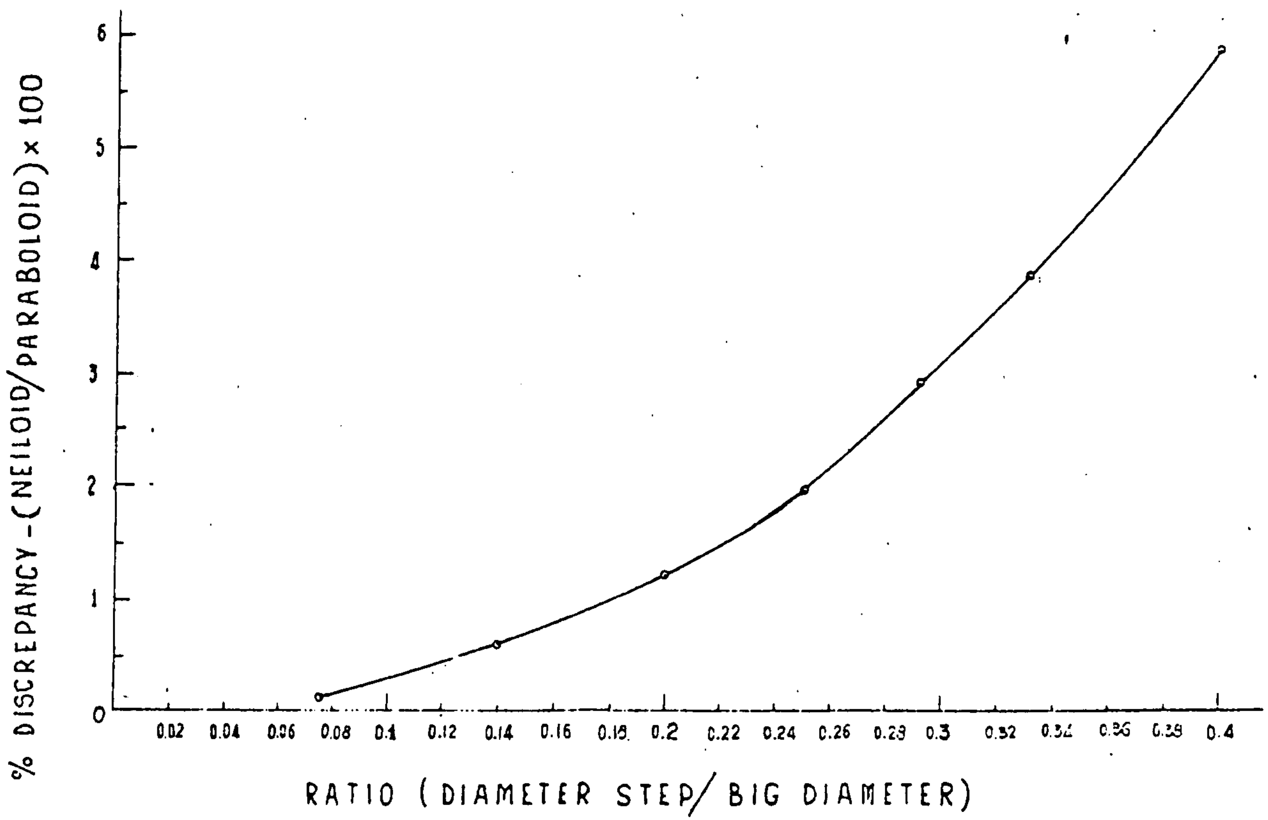
1	2	3	4	5	6	7	8	9	10
Dcm	dcm	Hm	VOLUME OF CONOID m^3	VOLUME OF PARABOLOID m^3	VOLUME OF NEILOID m^3	VOLUME DIFFERENCE m^3		VOLUME DIFFERENCE %	
						PARABOLOID-CONOID	NEILOID-CONOID	PARABOLOID-CONOID	NEILOID-CONOID
32.5	30.0	1	0.0767	0.0768	0.0767	0.0001	-0.0000	0.11	-0.05
35.0	30.0	1	0.0831	0.0834	0.0829	0.0003	-0.0002	0.39	-0.20
37.5	30.0	1	0.0898	0.0905	0.0894	0.0007	-0.0004	0.82	-0.41
40.0	30.0	1	0.0968	0.0981	0.0962	0.0013	-0.0006	1.34	-0.68
42.5	30.0	1	0.1042	0.1062	0.1031	0.0020	-0.0010	1.94	-0.99
45.0	30.0	1	0.1119	0.1148	0.1104	0.0029	-0.0015	2.60	-1.32
47.5	30.0	1	0.1199	0.1239	0.1179	0.0040	-0.0020	3.29	-1.64
50.0	30.0	1	0.1282	0.1334	0.1256	0.0052	-0.0026	4.00	-2.06
32.5	30.0	2	0.1534	0.1536	0.1533	0.0002	-0.0001	0.11	-0.05
35.0	30.0	2	0.1661	0.1666	0.1658	0.0007	-0.0003	0.39	-0.20
37.5	30.0	2	0.1796	0.1810	0.1788	0.0014	-0.0007	0.82	-0.41
40.0	30.0	2	0.1936	0.1962	0.1923	0.0026	-0.0013	1.34	-0.68
42.5	30.0	2	0.2083	0.2124	0.2063	0.0041	-0.0020	1.94	-0.99
45.0	30.0	2	0.2237	0.2296	0.2208	0.0059	-0.0029	2.60	-1.32
47.5	30.0	2	0.2397	0.2478	0.2357	0.0080	-0.0040	3.29	-1.69
50.0	30.0	2	0.2564	0.2669	0.2512	0.0105	-0.0052	4.00	-2.06
32.5	30.0	3	0.2301	0.2303	0.2300	0.0002	-0.0001	0.11	-0.05
35.0	30.0	3	0.2492	0.2502	0.2487	0.0010	-0.0005	0.39	-0.20
37.5	30.0	3	0.2693	0.2716	0.2682	0.0023	-0.0011	0.82	-0.41
40.0	30.0	3	0.2904	0.2934	0.2885	0.0039	-0.0019	1.34	-0.68
42.5	30.0	3	0.3125	0.3187	0.3095	0.0061	-0.0031	1.94	-0.99
45.0	30.0	3	0.3356	0.3444	0.3312	0.0088	-0.0044	2.60	-1.32
47.5	30.0	3	0.3596	0.3716	0.3536	0.0120	-0.0060	3.29	-1.69
50.0	30.0	3	0.3846	0.4003	0.3768	0.0157	-0.0078	4.00	-2.06
32.5	30.0	5	0.3835	0.3839	0.3833	0.0004	-0.0002	0.11	-0.05
35.0	30.0	5	0.4154	0.4170	0.4146	0.0016	-0.0008	0.39	-0.20
37.5	30.0	5	0.4489	0.4526	0.4471	0.0037	-0.0018	0.82	-0.41
40.0	30.0	5	0.4841	0.4906	0.4841	0.0065	-0.0032	1.34	-0.68
42.5	30.0	5	0.5209	0.5311	0.5158	0.0102	-0.0051	1.94	-0.99
45.0	30.0	5	0.5593	0.5740	0.5519	0.0147	-0.0073	2.60	-1.32
47.5	30.0	5	0.5994	0.6194	0.5894	0.0200	-0.0100	3.29	-1.69
50.0	30.0	5	0.6411	0.6672	0.6260	0.0262	-0.0131	4.00	-2.06

TABLE 5

VOLUME DISCREPANCY (NEILOID/PARABOLOID) % AND RATIO OF DIAMETER STEP/BIG DIAMETER

1	2	3	4	5	6	7	8	9
Dcm	dcm	Hm	VOLUME OF CONOID m^3	VOLUME OF PARABOLOID m^3	VOLUME OF NEILOID m^3	DIAMETER STEP cm	RATIO DIAMETER STEP/D	$100x(\frac{\text{column 6}}{\text{column 5}})$
32.5	30.0	1	0.0767	0.0768	0.0767	2.5	0.076	99.8
35.0	30.0	1	0.0831	0.0834	0.0829	5.0	0.142	99.4
37.5	30.0	1	0.0898	0.0905	0.0894	7.5	0.200	98.7
40.0	30.0	1	0.0968	0.0981	0.0962	10.0	0.250	98.1
42.5	30.0	1	0.1042	0.1062	0.1031	12.5	0.294	97.1
45.0	30.0	1	0.1119	0.1148	0.1104	15.0	0.333	96.1
47.5	30.0	1	0.1199	0.1239	0.1179	17.5	0.368	95.1
50.0	30.0	1	0.1282	0.1334	0.1256	20.0	0.400	94.1
32.5	30.0	3	0.2301	0.2303	0.2300	2.5	0.076	99.8
35.0	30.0	3	0.2492	0.2502	0.2487	5.0	0.142	99.4
37.5	30.0	3	0.2963	0.2716	0.2682	7.5	0.200	98.7
40.0	30.0	3	0.2904	0.2934	0.2885	10.0	0.250	98.1
42.5	30.0	3	0.3125	0.3187	0.3095	12.5	0.294	97.1
45.0	30.0	3	0.3356	0.3444	0.3312	15.0	0.333	96.1
47.5	30.0	3	0.3596	0.3716	0.3536	17.5	0.368	95.1
50.0	30.0	3	0.3846	0.4003	0.3768	20.0	0.400	94.1
32.5	30.0	5	0.3835	0.3839	0.3833	2.5	0.076	99.8
35.0	30.0	5	0.4154	0.4170	0.4146	5.0	0.142	99.4
37.5	30.0	5	0.4489	0.4526	0.4471	7.5	0.200	98.7
40.0	30.0	5	0.4841	0.4906	0.4841	10.0	0.250	98.1
42.5	30.0	5	0.5209	0.5311	0.5158	12.5	0.294	97.1
45.0	30.0	5	0.5593	0.5740	0.5519	15.0	0.333	96.1
47.5	30.0	5	0.5994	0.6194	0.5894	17.5	0.368	95.1
50.0	30.0	5	0.6411	0.6672	0.6280	20.0	0.400	94.1

Fig. 9. RELATIONSHIP BETWEEN VOLUME DISCREPANCY (NEILOID/PARABOLOID), %
AND RATIO (DIAMETER STEP/BIG DIAMETER)



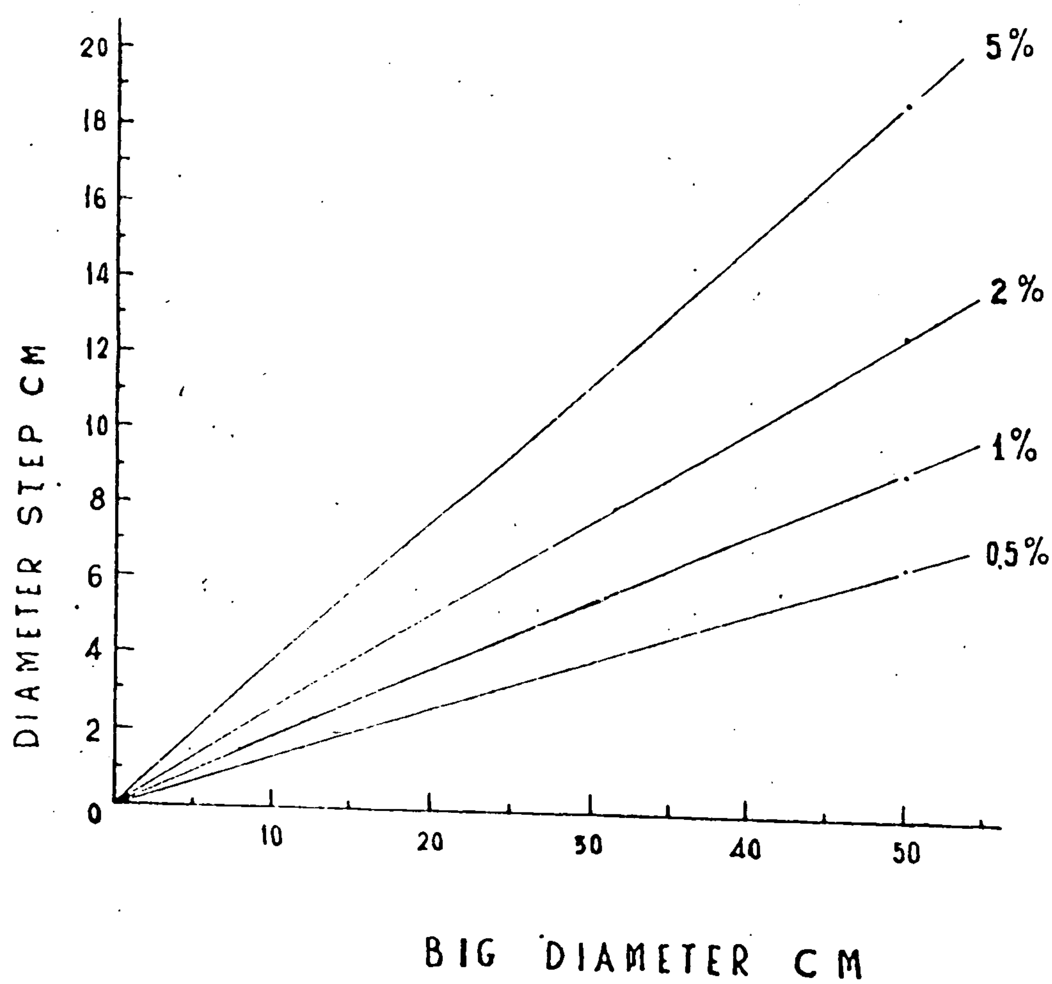


Fig. 10. Diagram for the selection of the diameter step

step of 4.5 cm should be used. If on the other hand one is content with a maximum discrepancy of 2% then it can be seen from Fig.10 that the required diameter step is 6.5 cm. The selection of the diameter step from Fig.10 according to the DBH of the stem will be the starting point for taking the sectional measurements for the volume estimation. Thereafter the measurer will need to reajust each subsequent diameter step for the remainder of the measurements up the stem as it will be dictated by Fig.10 in order that the same predefined accuracy be retained.

Following the above method (i.e. estimating the proper diameter step from Fig.10)sectional volume for any part of the tree stem can be estimated at a predefined degree of accuracy. This volume estimation is independent of the volume formula to be used, considering the degree of discrepancy as adequate for each particular situation. One of the benefits of the application of diameter step is that the possibility of coincidence with nodes or other irregularities of the stem (i.e. sudden reductions in diameter) is reduced compared with taking diameter measurements at tenths of total height. Also application of the diameter step establishes a better consistency among different data collectors.

Therefore for a consistent sectional volume estimation of a tree, one has to consider the following points:

1. Diameter steps should be used instead of standard lengths for diameter measurements.

2. The diameters will be measured at mid-internodal positions on the stem.

3. The appropriate diameter step should be used and this will be read from Fig. 10 for the fulfilment of the requirements of each particular situation. Among other possible uses of this method the following can be mentioned:

1. Diameter steps can be used as a criterion for checking previous volume estimations of tree sections where different formulae have been used and in each particular situation to qualify the volume results.

2. This method can be applied for volume estimation of either standing or felled trees, whether using an optical instrument or a caliper for diameter measurements.

3. In cases where an external factor, such as fertilization or thinning, has upset the prevailing growth conditions with possibilities of changing the stem form, or when there are doubts concerning the application of of existing volume Tables, the application of this method is recommended - and particularly in its strict form using a 2.5 cm diameter step - for volume estimation of the trees.

Finally it might be said that the method involves some extra work since the measurer needs to readjust the diameter step for total volume estimation of the stem, in order to retain the same degree of accuracy, but when one is faced with the problem of stem volume estimation in "sensitive" situations such as in different treatments that might affect stem form, the application of this method is still valuable.

This method was applied as a "check method" in this study in the sense of examining the difference between the two end diameters of each stem section (diameter step) It was found that all these differences were smaller than 2.5 cm and consequently any of the three formulae of page 43 could be used

CHAPTER 5

EXAMINATION OF THE INTERNAL PATTERN OF GROWTH

5.1 INTRODUCTION

In this Chapter the internal pattern of growth of the trees as well as the response to fertilization are examined and presented. The growth pattern is examined in terms of graphs constructed with the help of the computer or by hand. In these graphs the growth pattern is examined in terms of mean annual ring width based on four radii or in terms of mean cross sectional area (contour diagram). The ring increment pattern is examined in several positions over the part of the stem where the discs were cut from successive internodes.

Statistics and other details concerning the stand characteristics will be given in Chapter 6 , together with the statistical analysis of the data. The internal pattern of growth is presented first to facilitate the understanding of external changes in the dimensions of the trees.

5.2 METHODS OF PRESENTATION

It is the usual practice when estimating or presenting the increment of the trees to measure ring widths of successive years at a height of 1.3 m distance from the ground level. There have been reports in the past (Fayle and Mcdonald, 1977) that ring measurements at breast height may be misleading owing to the possibility of marked shifts of increment along the stem. Hence, in this study, ring width patterns were studied at various heights not only in radial sections but longitudinally and radiolongitudinally to obtain a more detailed view of the distribution of increment throughout the stem.

So far there have been reports of different behavior of response to fertilization of trees belonging to different dominance classes (Mitchell and Kellog, 1972; Gessel <u>et al</u>, 1969). Because of the difference in behaviour of response, and since it is desirable to examine the response of the different tree classes, it was decided to present a series of graphs with examples of trees from the following dominance classes:

1. Suppressed and subdominant
2. Co-dominant
3. Dominant

With the help of the plotting facilities of the computer, diagrams presenting the internal structure of annual rings were plotted for all trees. Fig. 11 is a typical example of the basic tree diagrams from which information has been obtained to prepare subsequent diagrams. In Fig. 11 the pith of the tree is shown running diagonally on the left-hand side of the diagram and on the right, parallel to the axis of ordinates is the outer part of the tree(bark). The position of each annual internodal ring is represented by a short line and the width of the ring is represented by the length of this line. The three ring sequences, suggested for growth studies by Duff and Nolan (1953), can be seen in the tree diagram (Fig. 11) and are described as follows:

Ring sequence no.1

All the rings on one line running parallel to the vertical axis. This shows the change in ring width with height for increment laid down in a given year. The progression in this sequence is not only longitudinal but it is radial also.

Ring sequence no.2

All the rings parallel to the horizontal axis. These ring sequences are those which are mainly used for tree growth studies. In these sequences the progress is radial and from one year's growth to another, but in the same internode e.g. see Fig. 11. This is the sequence of rings which are measured on a section or on a core taken for instance from the BH position.

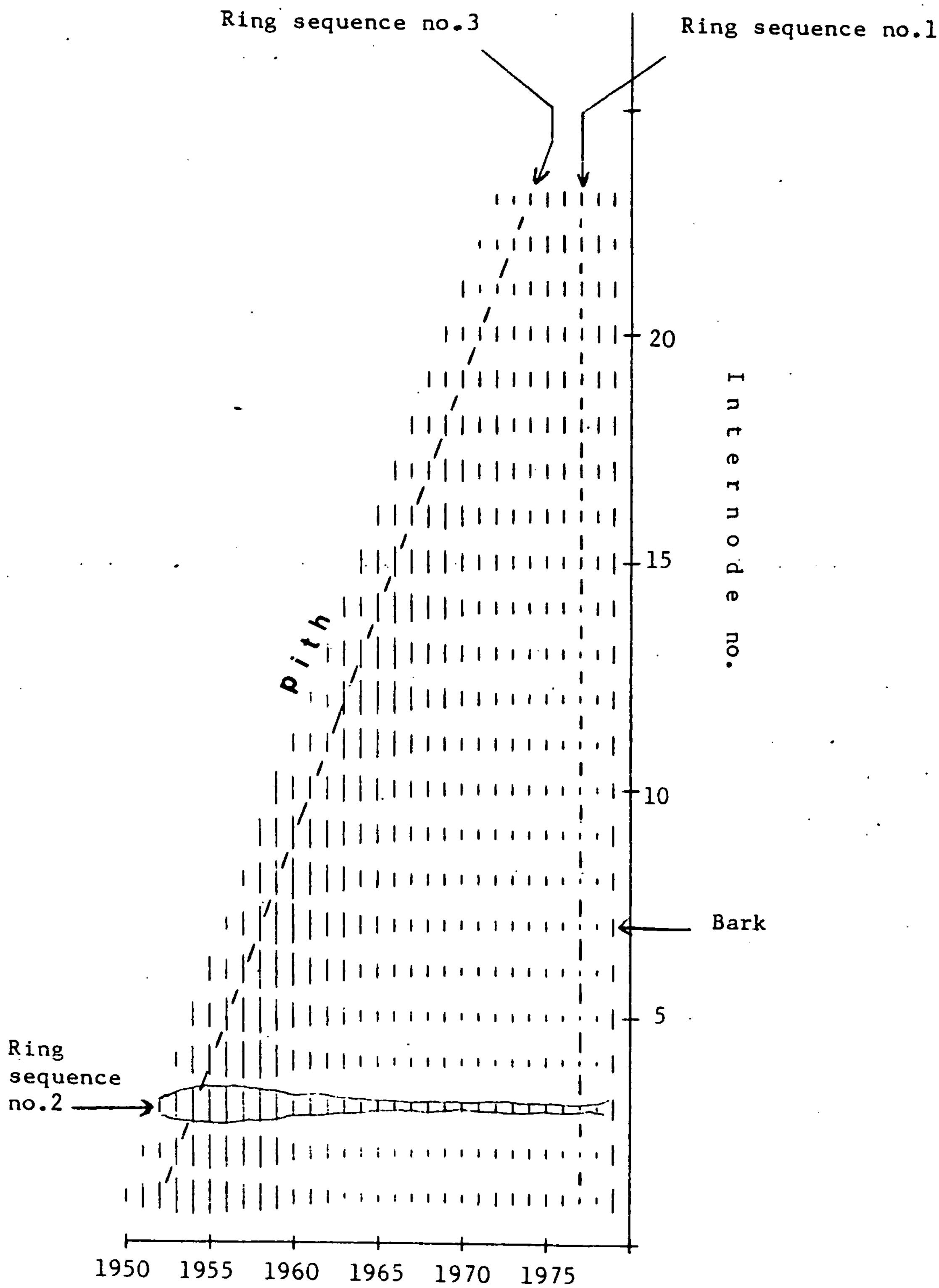


Fig. 11. DIAGRAM OF THE INTERNAL STRUCTURE OF A CONTROL DOMINANT TREE

Ring sequence no.3

All the rings in the line running parallel to the pith of the tree. The internodal rings which comprise any given sequence of this type have as a constant common element a uniform number of years separating them from the pith. For example all the fifth rings from pith comprise a no.3 sequence. Each no.3 sequence is laid down in internodes which are of uniform age. In these sequences the radial componenet which is the main characteristic of the two previous sequences has been eliminated. Therefore the main reason causing the characteristic pattern in both ring sequences no.1 and no.2 has been eliminated (Duff and Nolan, 1953). Ring sequences no.3 can be used as an index of the prevailing conditions of site and stand density in the forest (Richardson 1961, Duff and Nolan 1953).

Contour Diagrams (Figs. 12 & 13)

Finally an alternative graphical method is used to present those results which make it easier to appreciate the differences between:

a. Dominance classes and

b. Treatments

This method was also used for ring width as well as ring area. The method involves averages for trees of each dominance class and treatment, and uses contour diagrams to portray the information given in Fig.11 . As in the previous diagrams the line running diagonally on the left hand side represents the pith and on the right hand side the perpendicular line represents the outer part of the tree. Inside the diagrams there are lines enclosing zones of the stem with a particular range of ring area or ring width. Each of these zones has been shaded using different patterns and colours. Such diagrams were prepared for all tree dominance classes and treatments using the computer plotting facilities.

5.3 CONTROL TREES

The distribution of ring area as well as of ring width over the stem of the control trees, are presented and examined in contour diagrams (Figs. 12 & 13). References are also made to the plots for ring sequences no.1, no.2 and no.3

5.3.1. Ring area distribution (Figs. 12a, 12b, 12c)

The diagrams of Fig. 12 represent averages for the three dominance classes:

12a represents the dominant class

12b represents the co-dominant class and,

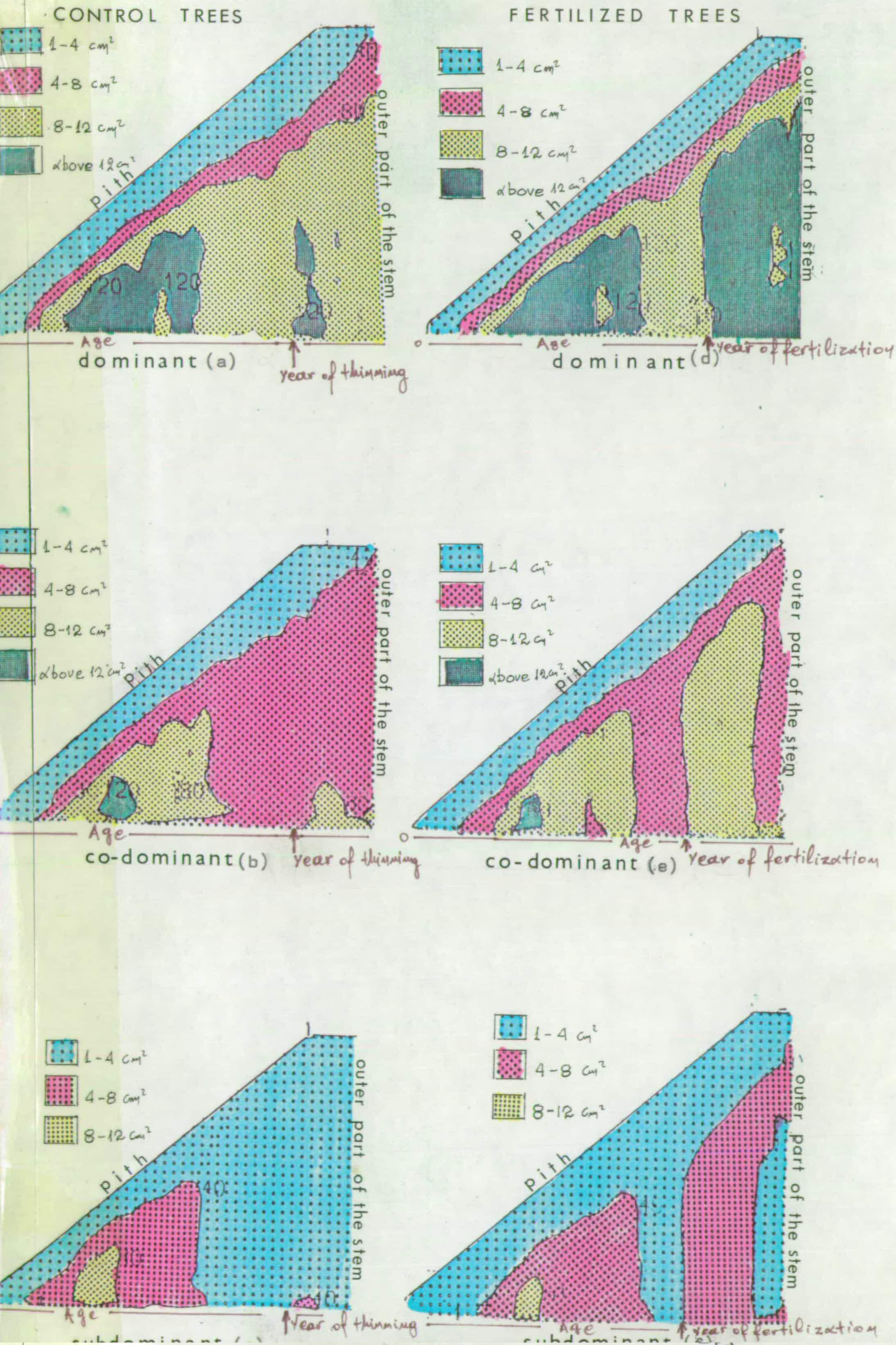
12c represents the suppressed and subdominant class

Examination of the cross sectional area distribution reveals that during the early stage of the life of the trees (lower left part of the diagram) ring area increases from top to bottom in all the dominance classes. The rate of growth is small in the suppressed-subdominant class, intermediate in the co-dominant one and far more pronounced in the dominant class. Despite this the pattern of the distribution is more or less the same in all classes since the trees were growing in the same stand and under similar soil and climatic conditions.

This pattern of growth is in agreement with Pressler's growth law (Larson, 1963) as well as with Assmann's report (1970). Pressler's growth law states that area increment at a particular point of the stem is proportional to the amount of foliage above this point. Assmann reports that in Norway spruce (*Picea abies*) cross sectional increment decreases above the base of the crown. Finally the same pattern of ring area distribution has been reported by Satoo, Nakamura and Senda (Larson, 1963).

Fig. 12

RING AREA DISTRIBUTION OVER THE STEM



Following thinning, trees of all classes responded in the same way, that is, by an increase of ring area in the lower part of the stem. This can be seen in Figs. 12a, 12b, 12c. Examining this response in the three classes it can be said that it was small in the trees of the suppressed subdominant class (12c), bigger in the do-dominant class (12b), and biggest in the dominant class (12a). For the last class it must be said also that the increase continued higher in the stem. The response of trees to the new conditions induced by thinning seem to have lasted less in the suppressed-subdominant class and more in the dominant one, with the trees of the co-dominant class showing an intermediate response.

The appearance of an increase in ring area in the lower part of the stem, following thinning, is in agreement with Metzger's theory (Larson, 1963) and Gray's theory (1956), both based on the idea that the stress created by the wind on the crown was propagated downward and culminated in a maximum at the stem base. Hence strengthening of the stem, in terms of increased cross sectional area, paralleled the stress gradient downward in the stem. Similar reactions to mechanical stresses have been investigated and confirmed by Assmann, Siostrzonek and Zahn (Assmann, 1970) in thinned stands of spruce (*Picea abies*), and Assmann concluded that:

> "reactions to mechanical stresses appear to be of such overwhelming importance that they have a decisive influence on the development of stem form".

In this study the limited evidence from the sample data presented in the previous form suggests that changes in growth distribution following thinning are in agreement with the theory of the wind acting as one of the factors of stem form development.

5.3.2. Ring width distribution

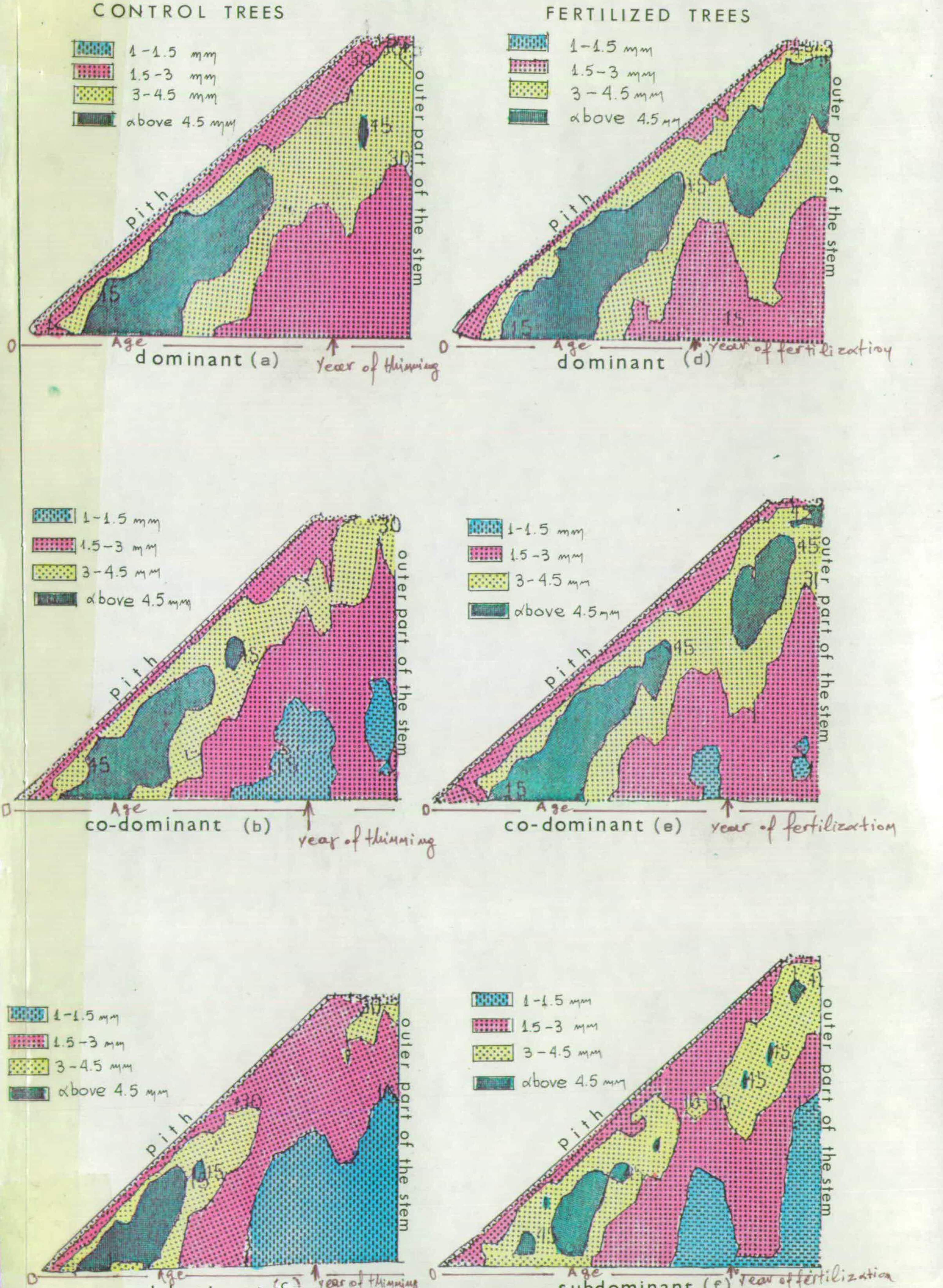
In Fig. 13 the diagrams a,b and c again represent the average trees of the three dominance classes as they were described in the previous section, but this time they present the ring width distribution. It can be seen again that in the early stages of the life of trees (lower left hand part of diagrams) all the trees had a higher rate of growth which was probably the result of better growing conditions due to a lack of competition before the crown closure of the stand. It can also be observed that they were following the same pattern of growth. In the examination of ring width distribution since references are made apart from the contour diagrams to the plots for ring sequences no.1, no.2 and no.3, the three dominance classes are examined separately:

Suppressed-subdominant class (Fig. 13c)

As can be observed from a visual inspection of the diagrams of Fig. 13, suppressed and subdominant trees had a lower rate of growth than the trees of the other two classes, it can be seen in Fig. 13 that the zone with the higher rate of growth - in terms of ring width - is in the lower left hand part of the diagram which in terms of age corresponds with the early years of the trees life. After that period ring width starts declining from pith outwards. Examining the same diagram from top to bottom we can also see the development of a small zone of increased ring width in the upper right hand part of the diagram (upper part of the tree). This was probably the result of thinning. Examination of the ring sequences no.1 (Fig.14c) reveals the existance of a quite definite pattern of the ring width in the longitudinal direction. Ring width increases from the top of the tree to the base of the crown then declines to the bottom of the tree.

Fig. 13

RING WIDTH DISTRIBUTION OVER THE STEM



This declining pattern stabilizes lower with small variations.

In the ring sequences no.1 corresponding to the years following thinning there is an indication of a general increase for the period 1970-74, but what is more important is that the increment is distributed over the stem following the same pattern, with the maximum of the ring width appearing in the lower part of the crown. There is also an indication of a slight increase of ring width after 1970 (year of thinning) in these sequences at the bottom of the tree, a characteristic that was not clear in the corresponding diagram.

In the ring sequences no.2 (radial direction) of Fig.15c ring width follows also a definite pattern, the main feature of which is the characteristic increase and decrease of ring width from pith outwards. Examination of these sequences in the lower part of the diagram indicates a slight tendency in the ring width to increase gradually after the year 1970, the increase being bigger in the upper part of the diagram.

Finally in Fig.16c examination of the ring sequences no.3 indicates a slight downwards trend in ring width from the year 1958 to 1971. Following thinning (in 1970) an upwards tendency begins which lasts until 1974.

Co-dominant class (Fig. 13b)

In this Figure the situation during the early stages of trees life - in terms of ring width - is similar with the one previously described (lower left hand part of the diagram). Later on, as canopy closes and competition initiates there appear zones of smaller ring width but they are still of a higher rate than the corresponding ones in the previous class.

Following thinning there is an increase in ring width in the

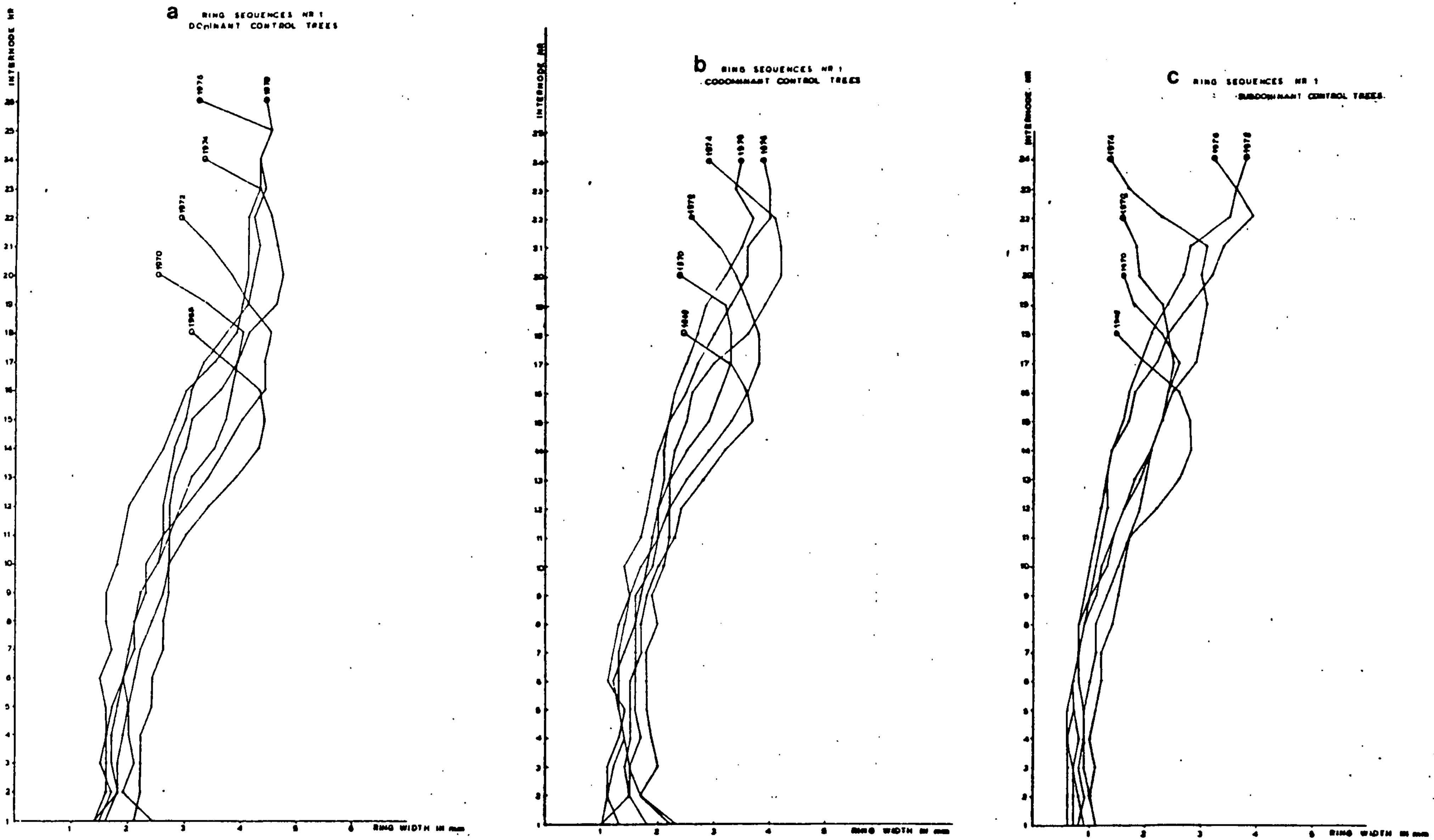
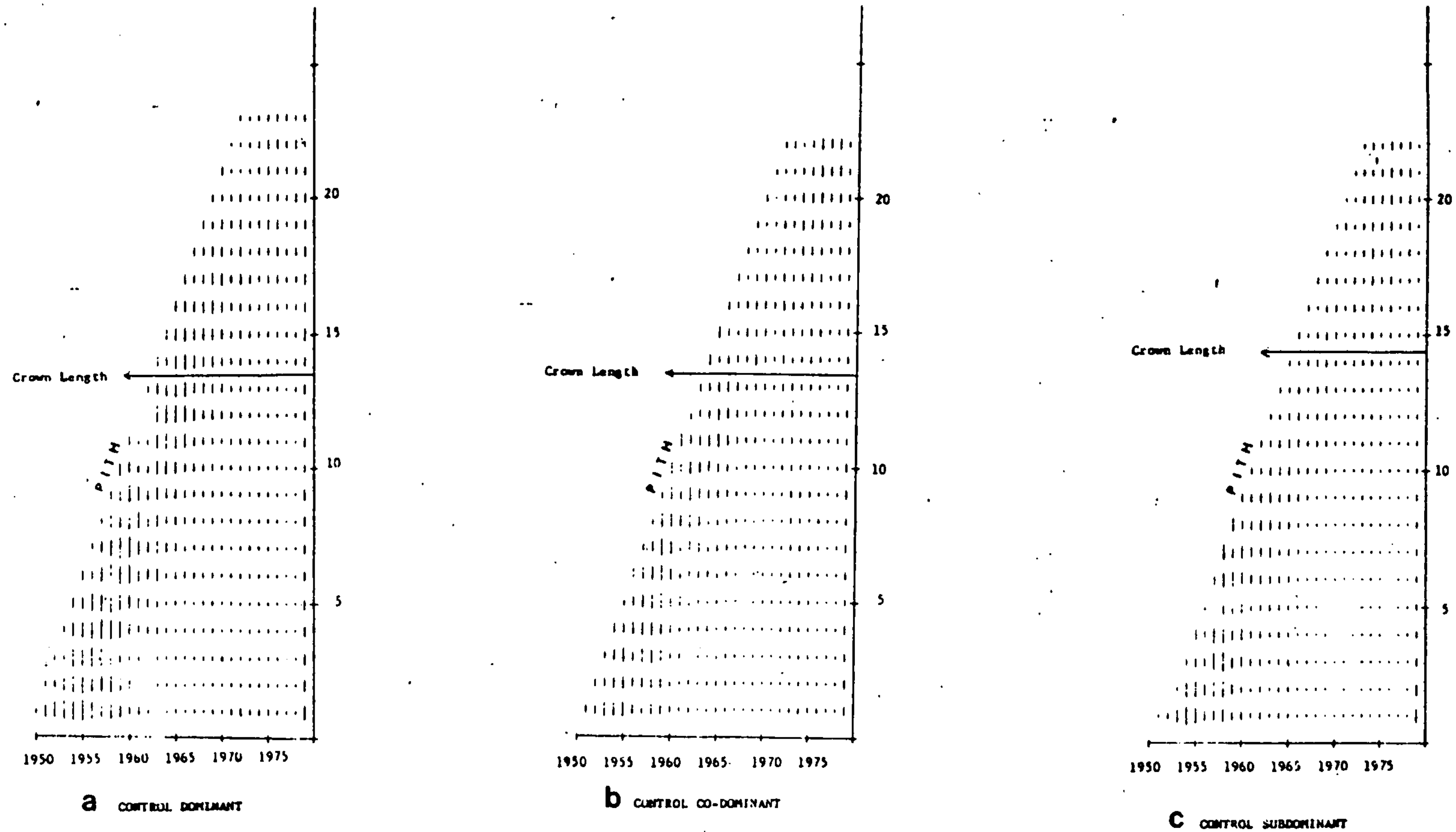


Fig. 14 Mean
Ring Sequences No. 1, a dominant, b co-dominant, c-subdominant
Control trees



<u>Fig. 15</u> Mean Ring Sequences No. 2, a dominant, b co-dominant, c subdominant Control trees

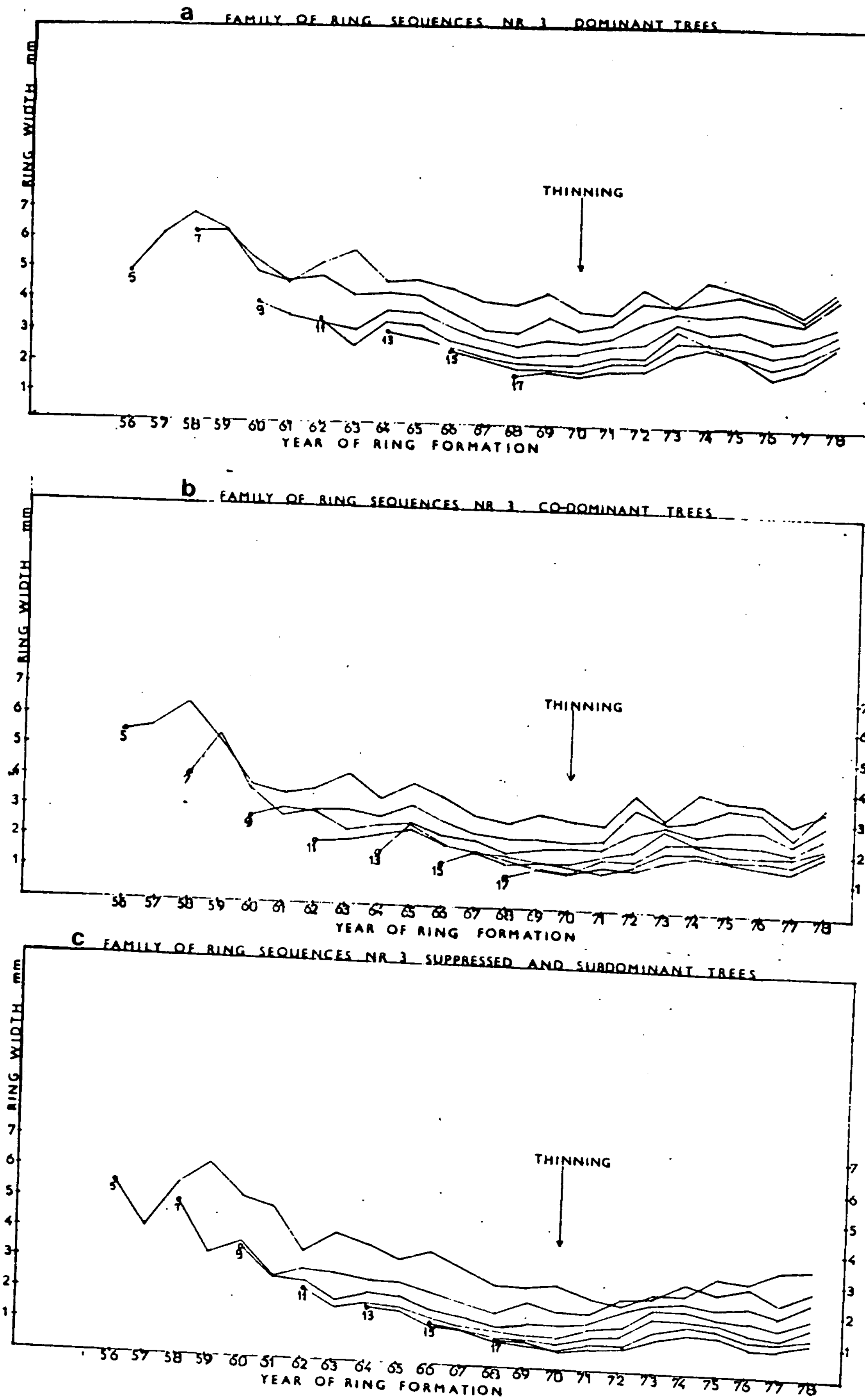


Fig. 16 Mean Ring Sequences No. 3, a dominant, b co-dominant, c subdominant

Control trees

upper part of the trees (upper right hand part of the diagram). Later on ring width starts declining again and a zone of smaller ring width appears in the middle/lower part of the stem but below this the previous pattern continues. The same features of ring width appear in the no.1 ring sequences in Fig.14b, as they were described in the contour diagram of this class. In the no.1 sequences a slight increase in ring width following thinning is also indicated at the bottom of the tree. In the radial direction - ring sequences no.2 -Fig.15b, the same pattern, as described in the previous class, repeats itself and following thinning there is an increase of ring width in the lower part of the tree as well as in the top. This time the increase is more pronounced than in the case of the suppressed-subdominant class. Finally in the ring sequences no.3, Fig.16b, the same trends as in the previous class appear before as well as after thinning, but the upwards increase after 1970 is bigger than in the suppressed-subdominant class.

Dominant class (Fig. 13a)

In the contour diagrams of this figure there is a repetition of the growth distribution in terms of ring width, as previously described, but the rate of growth is the biggest among the classes examined so far. Following thinning there appears to be a broader zone of bigger ring width extending downwards (upper right hand part of the diagram). Inside there is a small area of above 4.5 mm ring width, a characteristic which was absent in the previous two classes. In the ring sequences no.1, no.2 and no.3, Figs 14,15/16(a) the same situation is presented reflecting the new stand density conditions. One distinguishable characteristic in all the three ring sequences is the difference in ring width, as compared with that of the corresponding two previous classes, which reflected the dominant position of the trees of this class in the stand.

5.3.3. Summary

Summarizing after the examination of growth distribution over the stem of the control trees in terms of ring area and ring width, using either contour diagrams and/or ring sequences the following points are worth noting:

1. After the year of thinning there was an improvement in the growth conditions prevailing in the stand, due probably to lesser competition in combination with an increase in crown efficiency. This was expected since a smaller number of trees - after thinning - were sharing the same amount of nutrients on the same area.

2. In the contour diagrams, in terms of ring width, the response to thinning was reflected in the development of zones of bigger ring width in the upper part of the trees, as well as in a downwards extention of these zones towards the middle part of the stem. In terms of ring area these diagrams showed the response reflected with the development of bigger ring area in the lower part of the tree stem.

3. Examination of the contour diagrams and/or ring sequences indicated that the trees responded to thinning in accordance with their position in the stand canopy that is dominant trees responded more than the co-dominants and the latter responded more than the suppressed-subdominant ones.

4. Ring sequences no.1 showed that the increment distribution, in terms of ring width, followed the same pattern from one year to the next in the longitudinal direction of the tree stem, with the maximum of the ring width always appearing in the lower part of the active crown. The response of thinning does not appear very clear

in these ring sequences although they indicated an increase of ring width all over the stem and a characteristic small increase in the bottom.

5. Ring sequences no.2 followed their characteristic pattern at any internode over the stem and they showed a characteristic increase following thinning in the lower part of the stem, indicating probably the results of the wind action.

6. In the ring sequences no.3 the slight decline in the growth conditions of the stand is indicated as a slight downward trend in the lines from 1958 to 1970. Despite small random yearly variations the general trends of these sequences is the same in all the tree classes. Following thinning there appears a slight upwards trend in the lines of the diagram of these sequences, probably denoting the improvement in site and density conditions (Duff and Nolan, 1953, Richardson, 1961).

7. Close examination of the diagrams of ring sequences no.1 and no.2 reveals that the response to thinning started after 1-2 years and lasted almost 4 years.

8. As it is indicated from the diagrams of ring sequences no.1 and no.2, a definite pattern exists in the ring width increment and distribution over the stem of Sitka spruce trees.

5.4. FERTILISED TREES

In 1970 two silvicultural treatments took place in the experimental area:

1. Fertilization
2. Thinning

Both these treatments might probably change the environmental conditions prevailing in the stand and thus affect the growth of the trees, by influencing their physiological activities. Hence having already examined the results of thinning, any differences between the previously examined group of thinned trees (control), and the group of trees of this paragraph (fertilized+thinned) must be attributed to fertilization. The same methods that were applied for the examination of ring width and ring area over the stem of the control trees, are applied correspondingly for the fertilized trees.

5.4.1. Ring area distribution, Figs. 12d,12e,12f

The diagrams d,e and f of Fig.12 represent averages for the three dominance classes:

12d : represents the dominant class

12e : represents the co-dominant class

12f : represents the suppressed-subdominant class

Examination of the ring area in the three contour diagrams reveals the existance of the same features in the pattern of development as in the corresponding case of the control trees during the early stage of life of the trees.

The situation changes after fertilization. Examining the diagrams from top to bottom we can see, that,following fertilization ring area starts increasing, reaches a maximum then continues at this level through the rest of the stem more or less constantly with a secondary increase in the bottom, characteristic of the trees of co-dominant and suppressed-subdominant classes. Comparison of the diagrams of the three classes reveals that the increased ring area more or less followed the same distribution over the stem of the co-dominant, Fig.12e, and suppressed-subdominant trees, Fig.12f, in the sense that some years later it started declining over the biggest part of the stem, apart from the bottom where it continued following the previous increased pattern. The trees of the dominant class showed the greater response as compared with the trees of the other two classes. This response was distributed all over the stem and continued without any declining pattern. The characteristic feature of Fig.12d is the big zone of the increased ring area. In terms of rate of growth among the three classes, suppressed and subdominant class had the smaller, co-dominant class had an intermediate, and dominant class had the bigger rate of growth.

Finally, comparison of the diagrams of the control (thinned) trees, Figs.12a,12b,12c with the corresponding ones of the fertilized (thinned) trees, Figs.12d,12e,12f indicates that the fertilized trees followed a different pattern of growth distribution, in terms of ring area. Ring area, following fertilization increased all over the tree stem in all dominance classes, while in the case of the control trees ring area increase only at the bottom of the trees.

5.4.2. Ring width distribution, Figs.13d,13e,13f

In Fig. 13 are presented average tree diagrams showing the ring width distribution over the stem of the three dominance classes:

13d : represents the dominant class

13e : represents the co-dominant class

13f : represents the suppressed-subdominant class

Since during the examination of ring width distribution over the tree stem, references, apart from the contour diagrams, are made to diagrams of ring sequences no.1, no.2 and no.3, the three dominance classes are examined separately.

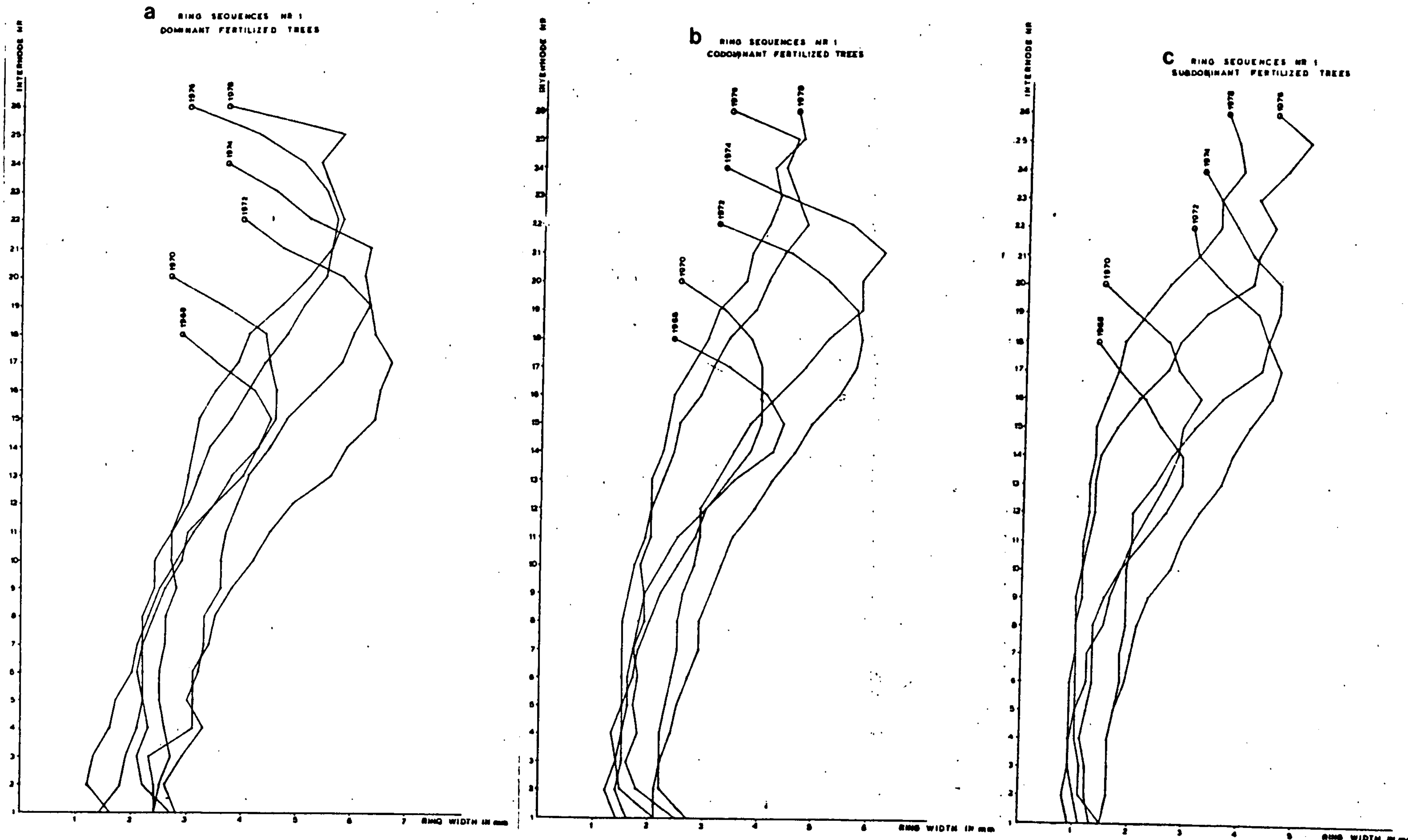
<u>Suppressed and subdominant class Fig.13f</u>

The trees of this class had a lower rate of growth since early years of their life, in comparison with the trees of the two other classes, in terms of ring width. Starting the examination of the internal structure of the average tree again, after the initial phase growth seem to be declining in the lower part of the tree stem and this is reflected in the minimization of ring width in the area of the central-bottom part of the stem. Ring width continuous to be bigger in the upper part of the stem.

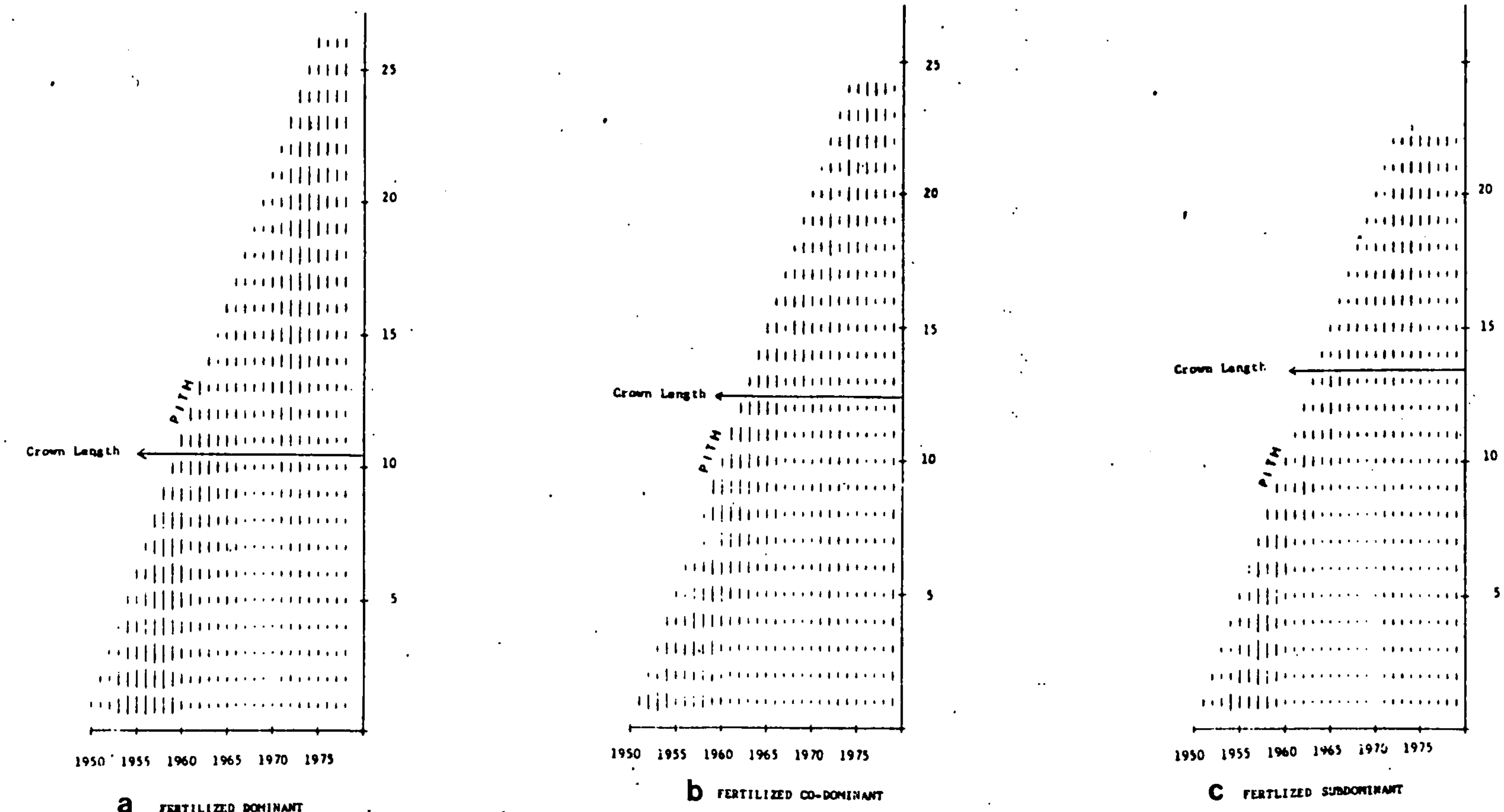
The situation changes following fertilization and zones of bigger ring width extend further down in the tree stem as it is indicated in the diagram f of Fig.13 . Smaller zones of bigger ring width are also developed in the upper part of the tree which is covered by the active crown. The same situation appears in the diagrams of no.1 as well as no.2 ring sequences, Figs.17c and 18c correspondingly. Examination of the ring sequences no.3 in Fig. 19c indicates a characteristic upwards trend after the year that fertilizers were applied, which is more pronounced than in the corresponding case of the control class (Fig.16c)

<u>Co-dominant class, Fig.13e</u>

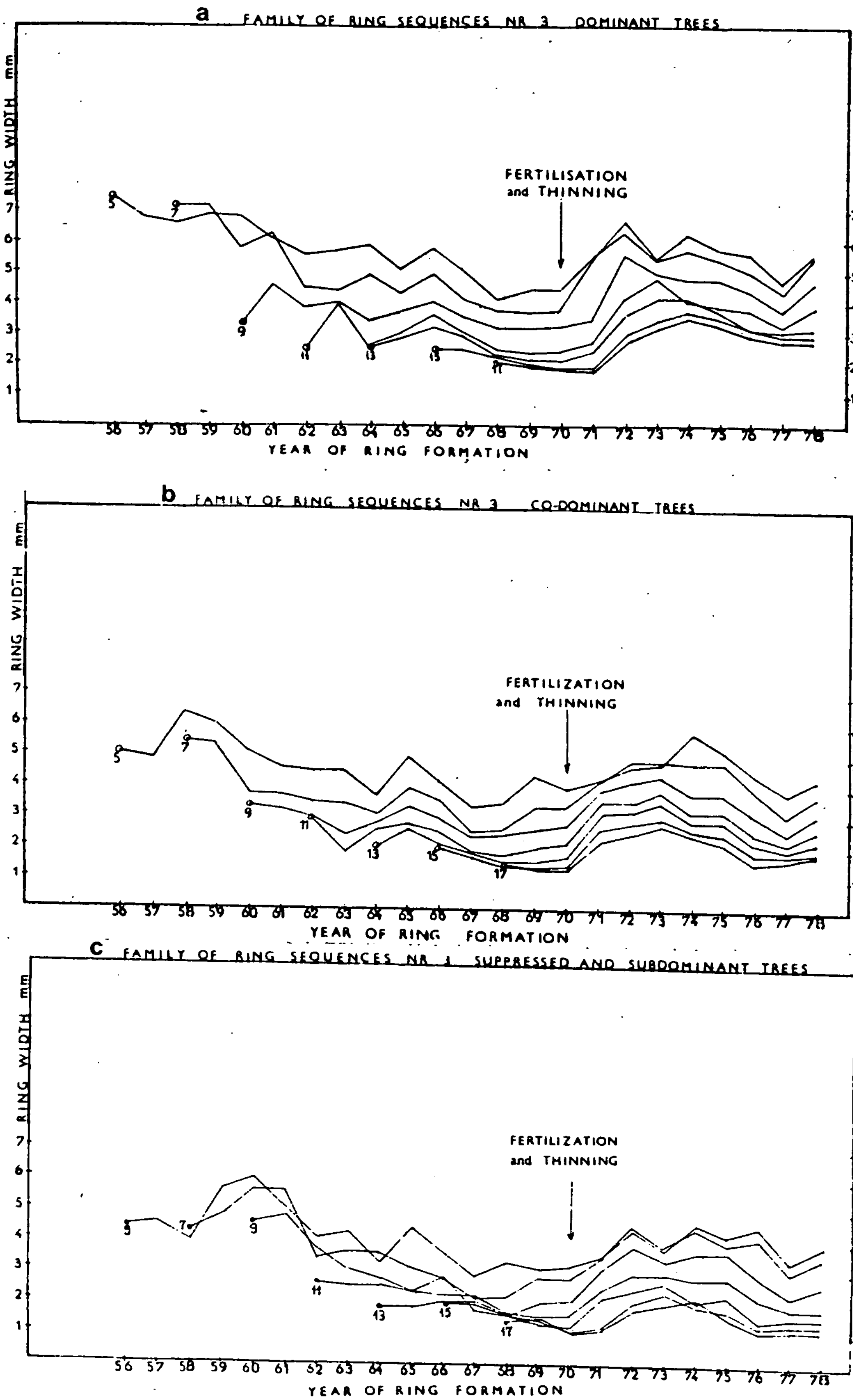
In this diagram the internal structure of the tree, up to the year that fertilizers were applied, seems to have more or less the same prevailing pattern as the corresponding control trees. This pattern changes following fertilization with the appearance of zones of



<u>Fig. 17</u> Mean Ring sequences No. 1, a dominant, b co-dominant, c subdominant Fertilized trees



<u>Fig. 18</u> Mean Ring Sequences No. 2, a dominant, b co-dominant, c subdominant
Fertilized trees



Mean
Fig. 19 Ring Sequences No. 3, a dominant, b co-dominant, c subdominant
Fertilized trees

bigger ring width in the upper portion of the stem. These zones of bigger ring width extend further down the tree stem in the case of the fertilized tree diagram as compared with the corresponding control. The rate of growth compared with that of the corresponding previous fertilized class seem to be higher.

The same situation is indicated in the ring sequences no.1 as well as in no.2 (Figs 17b, 18b) correspondingly. In the longitudinal direction ring sequences no.1 following fertilization ring width is bigger and with a more pronounced maximum as compared with the corresponding case of the controls. In the radial direction ring sequences no.2 after fertilization there follows a sharper increase in ring width as compared with the corresponding class of the controls. Finally in the ring sequences no.3 Fig.19b the general upwards trend is far more pronounced than in the corresponding case of the control class.

Dominant class, Fig. 13d

Comparison of this contour diagram with the corresponding one of the control class indicates again similarities up to the year of fertilization. Later, following fertilization the widest zone of increased ring width among the cases examined so far, control and fertilized, appears in the upper part of the tree diagram. Also, zones of increased ring width extend further down the tree stem and generally the higher rate of growth is apparent as compared with that in the control class.

In Figs. 17a, 18a the ring sequences no.1 and no.2 show in details the response to fertilization. In Fig.19a the upwards trend in the ring sequences no.3 is the sharpest so far. The duration of the response as well as its size, in terms of ring width, is also apparent in the same Figure.

5.4.3. Summary

Summarizing the response of the trees to fertilization, in either terms of ring width or ring area, the following points are worth noting:

1. Following fertilization the improved photosynthetic status of the trees, due to increase in leaf area and/or increased photosynthetic efficiency, provided conditions enabling the cambium to form cells of probably larger size, which were reflected to an increase in ring width.

2. As it appears from the contour diagrams, Figs. 12 and 13 fertilization increased growth all over the stem (in terms of ring area) in the trees of all classes.

3. In terms of ring width distribution, contour diagrams revealed that following fertilization the main feature of the growth pattern was the development of bigger ring width in the upper part of the tree extending downwards in the part of the stem covered by the crown.

4. In the longitudinal direction (ring sequences no.1) Figs.14 & 17 ring width appears to follow the same pattern repeated year after year. The maximum of ring width of the recent years (1968-1978) appears to be always in the lower part of the crown of the trees (Farrar, 1961). This maximum, following fertilization is more pronounced in the fertilized than in the control trees. After maximization, ring width starts decreasing gradually and in the case of fertilized trees seem to stabilize lower in the stem than in the case of the control trees. Finally after stabilization, the ring width in the lower part of the stem seem to be subjected to small random variations and there is evidence of a second increase at the bottom of the stem. This latter increase is probably a result

of the development of mechanical tensions in this part of the stem caused by the increasing size of the tree.

5. In the radial direction (ring sequences no.2) Figs. 15 and 18 the ring width follows a standard pattern similar to the one already described in the case of the control trees. After 1970 there appears to be a second increase in the ring width which lasts for about 3-4 years and is followed by a decrease. This increase appears in all the tree categories and is more pronounced in those trees in the upper part of the canopy. The maximum ring width appears in the third to fourth year following fertilization and is characteristically bigger than in the case of the control trees. After that ring width starts declining.

6. In the ring sequences no.3,Figs.16 and 19 the prevailing conditions in the stand are reflected clearly (Duff and Nolan,1953). Hence from 1960 onwards there is a slight decline in these ring sequences denoting a decline in the growth conditions in the stand; probably caused by a decrease in the existing amount of nutrients. This decline of ring width in these sequences can be seen in both control and fertilized trees from 1960-1970 since all the trees were growing in the same stand and therefore were subjected to the same general prevailing conditions of the site. After fertilization (in 1970) the upwards trend in these ring sequences denotes the response of the trees in the improved site conditions caused by the increased amount of nutrients provided by fertilization. This time the rate of increase in terms of ring width of the fertilized trees is far beyond the corresponding control trees.

7. The duration of the response to fertilization, as it is indicated from examination of ring sequences no.2 and no.3, ring sequences, seems to be 3-4 years.

8. Trees responded to fertilization in accordance with their position in the stand canopy, that is, the dominant trees responded more than the co-dominants and the later more than the suppressed-subdominant trees.

CHAPTER 6 EXAMINATION OF THE RESPONSES OF CROSS-SECTIONAL AREA, HEIGHT AND FORM FACTOR AS RELATED TO FERTILIZATION

6.1 INTRODUCTION

In the previous chapter we have already examined the effects of fertilization in terms of the internal pattern of increment. In the following paragraphs the insight gained from that analysis will help to establish sound criteria for the examination and estimation of tree responses in terms of directly measurable dimensions, such as basal area, cross sectional area at half of total height, total height, and indirectly in terms of volume. The response of form factor to fertilization will also be considered here, but the effect of fertilization on the external shape of the trees will be examined in the next chapter.

The response of the trees will be examined for a specified period of five years, i.e. 1970–75, since this was the period between two successive thinning operations carried out in the experimental area, during which trees were allowed to grow without any disturbance. The response will be examined in terms of:

1. Overall effects at the end of the above specified period (i.e. 1975)
2. Increment during the under examination period (i.e. (1970–75)

and will be accompanied by statistical tests and regression analyses.

Development of the response over time will also be examined here as well as the response of trees belonging to different dominance classes. Finally the response of volume/ha is examined in this chapter using different methods.

The following formulae were applied for the expression of the response in percent terms:

1. % overall response in 1975 = $100 \times (T_F - T_C) / T_C$ where

T_F = the value of the fertilized variable in 1975, and

T_C = the value of the control variable in 1975.

2. % response in increment = $100 \times (F_{INC} - C_{INC}) / C_{INC}$

where F_{INC} = fertilized increment during 1970-75

and C_{INC} = control increment during 1970-75.

The DBH frequency distribution of the sampled trees as well as of all the trees in the sampled plots for 1971 and 1975 is given in Appendix 2 . The statistical analysis of the results was carried out using the SPSS package (ERCC).

6 .2
CROSS SECTIONAL AREA RESPONSES RELATED TO FERTILIZATION :
BASAL AREA AND CROSS SECTIONAL AREA AT HALF OT TOTAL HEIGHT

In different silvicultural practices such as thinning or fertilization, the goal is to introduce a more favourable environment for the trees to grow. Previous research has found that such treatments may be beneficial in some cases but not in others. This difference is difficult to explain (Brix, 1976).

Working with different forest species several investigators reported increases in terms of DBH or BA growth in response to application of varying amounts of different fertilizers or combinations of them. Binns and Grayson (1967) referring to early fertilization experiments in Britain cited responses of 10 per cent in BA increment in mature Scots pine over a three year period following N fertilization and increases of 19 per cent in BA following the application of N plus P fertilization. From another experiment - again of mature Scots pine - they cited responses of 37 per cent in terms of BA increment following N fertilization over a three year period. For pole stage crops of Scots pine they also cited responses of 12 per cent in BA increment over a seven year period following N fertilization and 19 per cent following P fertilization. Brix and Ebel (1969) found that BA increased by 60 per cent in a pole stage Douglas fir stand over a three year period following N fertilization. Windsor and Reines (1973) found statistically significant differences in BA increment working with pole stage loblolly pine over a six year period following NP fertilization. Miller and Cooper (1973) working with pole stage Corsican pine in an N fertilization experiment over a seven year period found that "Response in terms of BA appeared in the year after the first application but was not accompanied by any changes in the pattern of growth throughout the season". They also reported that smallest trees showed a reduced response to the lower rates of fertilizer application. Finally, Whyte and Mead (1977) working with radiata pine in an NP fertilization experiment found a response of 12 per cent in BA over a five year period.

From all the above reports it is evident that fertilization can increase BA in different species and under certain conditions.

In this study having carried out stem analysis of the forty control and forty fertilized trees it was easy to calculate the under bark BA increment year by year(using the formula $G=\frac{\pi d^2}{4}$ and the diameter measurements described in chapter 3. However, before presenting the results following fertilization it was considered useful to present the BA of the sampled trees at the beginning of the experiment (1970) (Table 6)

TABLE 6

STATISTICS OF BASAL AREA AT THE BEGINNING OF THE PERIOD (1970)

Number of trees	Treatment	Year	Mean BA (m^2)	Standard Deviation	Variance	Standard Error	Range
40	Control	1970	0.0140	0.0051	0.000025	0.00080	0.020
40	Fertilized	1970	0.0141	0.0043	0.000016	0.00067	0.016

The entries of the above table indicate that control and fertilized trees initially had almost equal mean BA. Statistical analysis did not show any statistically significant difference between BA of the treated and control trees ($p<0.05$), but showed differences between blocks (Table1 , Appendix3).

EXAMINATION OF THE OVERALL EFFECTS IN BA

Five years later following fertilization, the situation changed and the corresponding statistics of BA are presented in Table 7.

TABLE 7

STATISTICS OF BASAL AREA AT THE END OF THE PERIOD (1975)

Number of trees	Treatment	Year	Mean BA (m^2)	Standard Deviation	Variance	Standard Error	Range
40	Control	1975	0.0174	0.0071	0.000049	0.0011	0.025
40	Fertilized	1975	0.0194	0.0062	0.000036	0.00098	0.026

The results of the above table indicate that at the end of the five year period there was a bigger increase in the mean BA of the fertilized trees which showed a response $0.002m^2$ or 11.5% if expressed in per cent terms of the mean BA of the control trees in 1975. In order to examine whether these differences were statistically significant or not, two-way analysis of variance was used (Table 2 , Appendix 3). The results of this analysis showed that these differences were not statistically significant ($p < 0.05$). The same analysis showed that the differences between blocks were significant ($p < 0.05$). We have already seen that such differences existed at the beginning of the experiment and probably they were the reason for non-existance of significant differences between treated and untreated trees, since they might be transferred and therefore contributed to the variability between blocks in 1975. This led to examination of BA differences in 1975 using analysis of co-variance. The most suitable covariate to be used for this purpose was considered to be the initial BA (BA in 1970). The results (Table 3, Appendix 3) showed this time that there were highly significant differences in BA of 1975 between treated and untreated trees ($p < 0.01$) and also there appeared to be no differences between blocks.

EXAMINATION OF BA INCREMENT DURING 1970-75

Next it was decided to examine the response in terms of BA increment between control and fertilized trees during the period 1970-75. BA increment during this period was 0.0034 m^2 for the control and 0.0053 m^2 for the fertilized trees (Tables 8a and b), and therefore giving a response of 0.0019 m^2, or if expressed in per cent terms almost 56% over the control increment during the same period. Analysis of variance of BA increment showed immediately (Table 4 , Appendix 3) that there was enough evidence to accept that the differences between treated and untreated trees were SS ($p < 0.01$) while at the same time there were no SS differences between blocks. These results were pointing to the direction that increment is a more sensitive variable to be used in such cases for the estimation of responses.

EFFECT OF DOMINANCE CLASS

In cases of application of different silvicultural treatments it is desirable to know the response of trees belonging to different dominance classes, since different patterns of response of trees of these classes might have repurcussions in terms of shading and suppression, and therefore might necessitate changes in the policy of thinnings (Miller and Cooper, 1973). In the next table is presented the mean BA increment as well as its response in the three dominance classes as they were defined in Chapter 2.

TABLE 8

MEAN BA INCREMENT AND RESPONSE OF TREES BELONGING TO THE THREE DOMINANCE CLASSES DURING 1970-75 (in m^2)

(a) CONTROL TREES (n=40)

Year	Mean BA of the suppr. and subdom. trees (n=12)	Mean BA of the codominant trees (n=14)	Mean BA of the dominant trees (n=14)
1970	0.0100	0.0123	0.0191
1975	0.0118	0.0153	0.0242
Increment	0.0018	0.0030	0.0030

(b) FERTILIZED TREES (n=40)

Year	Mean BA of the suppr. and subdom. trees (n=12)	Mean BA of the codominant trees (n=14)	Mean BA of the dominant trees (n=14)
1970	0.0097	0.0146	0.0171
1975	0.0125	0.0191	0.0245
Increment	0.0028	0.0045	0.0074

Dominance Class	Suppr.-subdominant	Co-dominant	Dominant
Response	0.0028-0.0018 = 0.0010	0.0045-0.0030 = 0.0015	0.0074-0.0051 = 0.0023

The results of the above table showed that dominant trees were responding more to fertilization than the co-dominants and the latter more than the suppr.-subdominant trees. Similar results in the same sequence of response of dominance classes have been reported by Windsor and Reines (1973). It seems apparent that trees assuming higher positions up in the canopy under competition, such as trees of dominant and co-dominant classes, are more efficient in using the nutrients provided by fertilization.

BASAL AREA INCREMENT AND RESPONSE DEVELOPMENT DURING 1970-75

Another desirable result in an analysis of the responses of trees to fertilization would be the development of the response in the years following fertilization to find the duration of the response as well as the year of maximum response. In Fig.20 is presented the mean BA development in terms of BA increment during 1965-75. In this figure it can be seen that both groups of trees had the same trend in BA development during 1965-70. Following fertilization in 1970 the two BA increment lines assume different trends.

In Table 9 as well as in Fig.21 is presented the development of the response of mean BA increment for each of the first five years following fertilization.

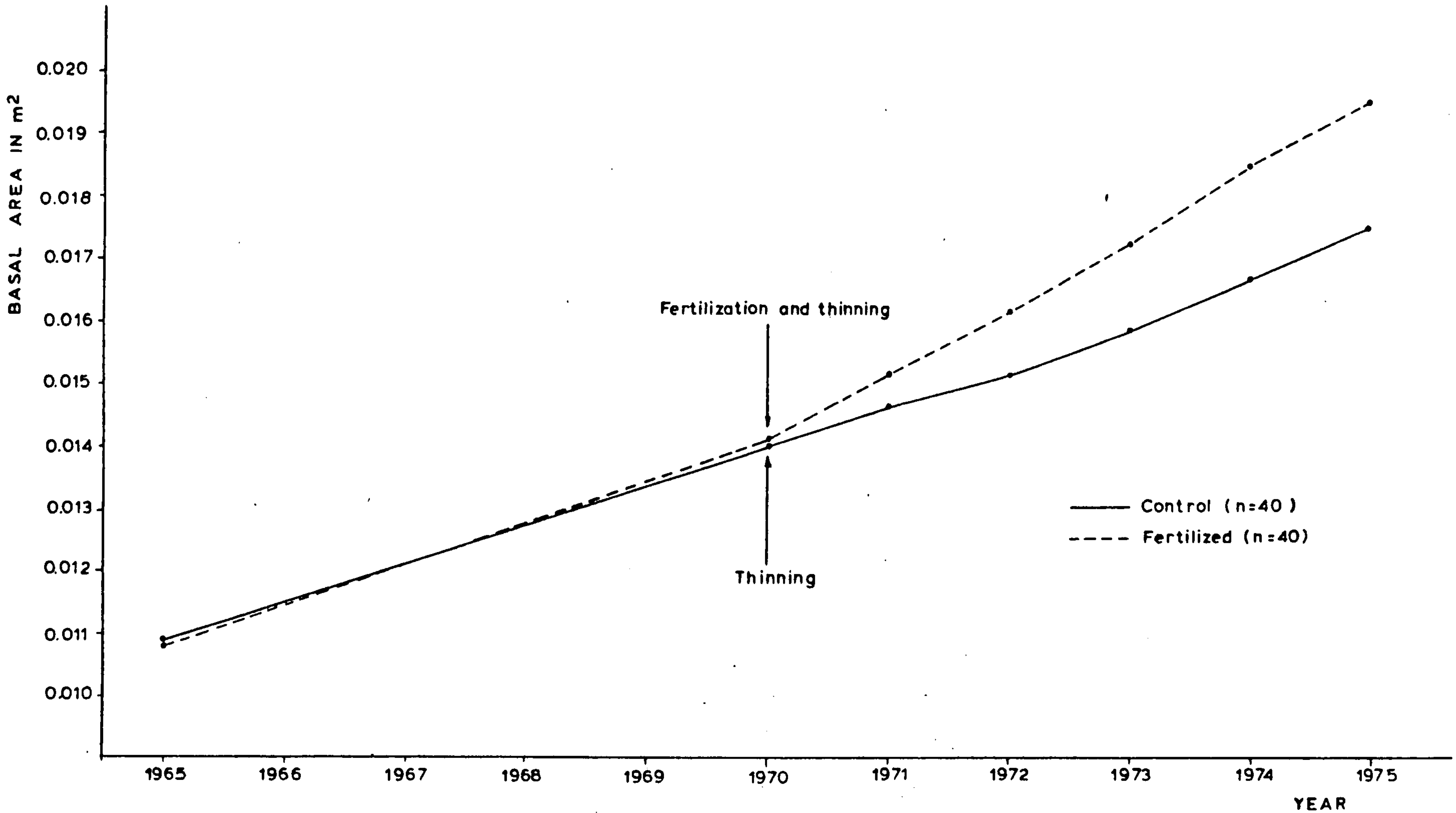


Fig. 20 Mean basal area development following fertilization

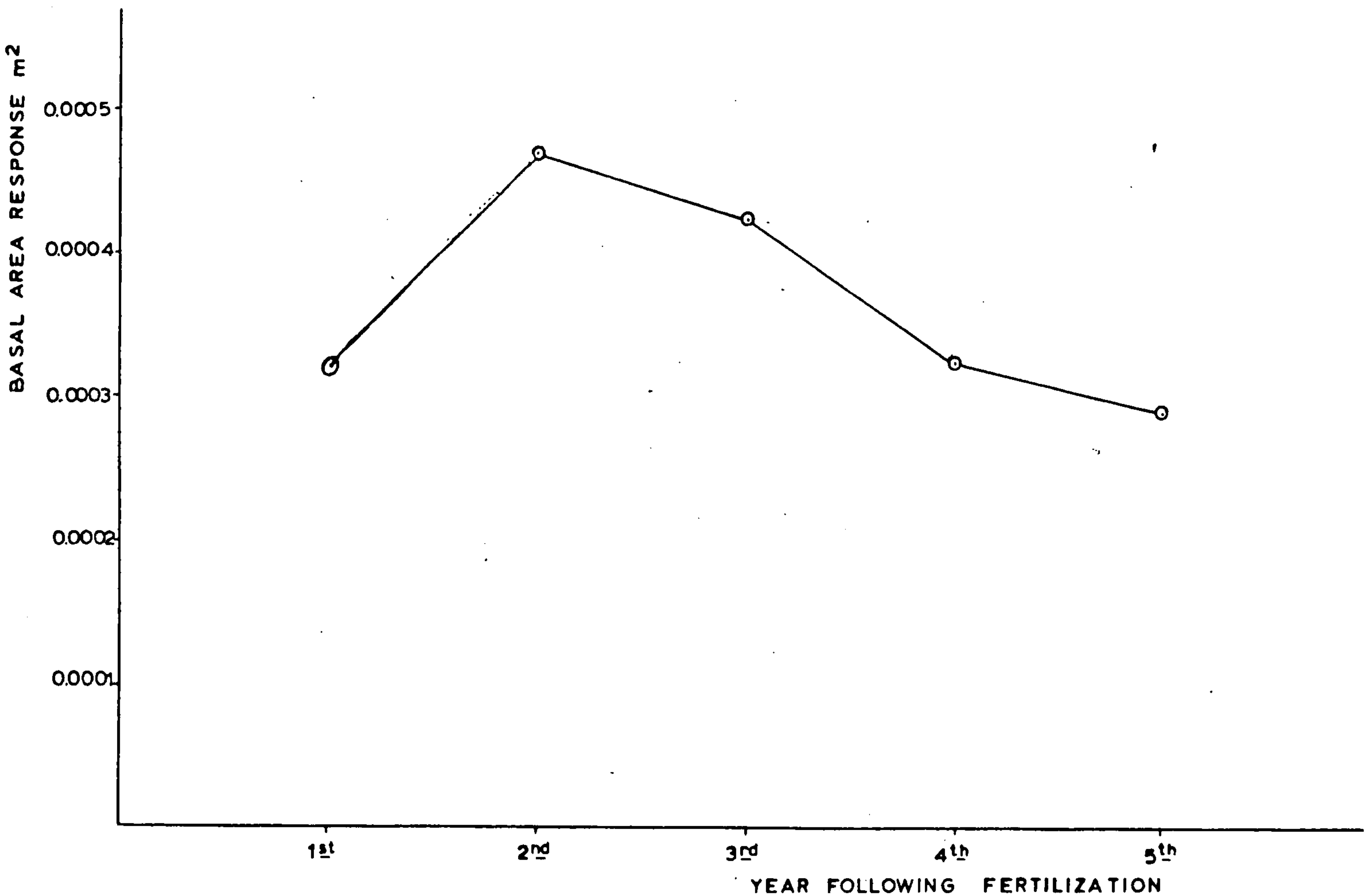


Fig. 21 Mean annual basal area response

TABLE 9

MEAN BASAL AREA RESPONSE FOR EACH OF THE FIRST FIVE YEARS FOLLOWING FERTILIZATION

Year since fertilization	Mean BA increment (m^2) Fertilized	Control	Response (m^2) (Fertilized incr.-control incr.)
1st	0.00091	0.00056	0.00035
2nd	0.00101	0.00055	0.00046
3rd	0.00110	0.00068	0.00042
4th	0.00113	0.00081	0.00032
5th	0.00108	0.00079	0.00029

From the results of the above table as well as from Fig.21 we can see that the maximum mean BA increment response to fertilization occurred during the second year after fertilisers were applied. In the third year the response started declining and finally in 1975 - five years since fertilization - the response reached a level well below that for the first year. The results of the above table also indicate that the response to fertilization started in the year of application, since both control and fertilized trees had almost equal mean BA (Table 6). Similar results as far as the appearance of response during the year of application of fertilizers have been reported by White (1956).

EXAMINATION OF CROSS-SECTIONAL AREA INCREMENT AT HALF OF TOTAL HEIGHT POSITION

For comparison with the above the increment response of cross-sectional area at half total height was analysed. From the stem analysis data the cross-sectional areas of the 40 control and 40 fertilized trees at half of total height were obtained for the years 1970 and 1975 using linear interpolation. Linear interpolation was considered adequate since the original diameter measurements had been taken on discs cut at each mid-internodal position.

In the next tables are presented the statistics of the cross-sectional area (CSA) at half of total height position for the years

1970 and 1975.

TABLE 10

STATISTICS OF CROSS-SECTIONAL AREA AT HALF TOTAL HEIGHT POSITION

Year	Number of trees	Treatment	Mean CSA m^2	Standard deviation	Variance	Standard error
1970	40	Control	0.00587	0.00228	0.0000052	0.000360
1970	40	Fertilized	0.00595	0.00185	0.0000034	0.000292
1975	40	Control	0.00776	0.00306	0.0000094	0.000484
1975	40	Fertilized	0.00925	0.00332	0.0000110	0.000525

The entries of the above table showed that in the beginning of the experiment both treated and untreated trees had almost equal cross-sectional areas. At the end of the five year period (1975) fertilized trees were benefited more than the controls by 0.00925–0.00776=0.00149m^2. The cross-sectional area increment of the control during 1970–1975 was 0.00776–0.00587=0.00189 m^2, while that of the fertilized trees was 0.00925–0.00595=0.00330 m^2 and thus giving a response of 0.00330–0.00189=0.00141 m^2 in terms of cross-sectional area increment during 1970–75 or 74% over the control increment during the same period. The corresponding response of basal area increment was 0.00190 m^2 or 56%.

Statistical analysis of cross-sectional area increment (Table 5 , Appendix 3) showed that the differences between treated and untreated were statistically significant ($p < 0.01$) and therefore contributing to the fact that cross-sectional area increment responses were taking place not only at breast height but also higher up the stem. This was in accordance with the results of the analysis of the internal pattern of growth in the tree diagrams (Chapter 5 , para.4.2).

EXAMINATION OF THE RELATIONSHIP BETWEEN INITIAL (1970) BA AND FINAL (1975) BA

What remained after the previous analyses was the examination of the relationship between initial BA and BA increment. This would enable to see whether trees increased in accordance with their initial BA. Regression analysis was used for this purpose, one for the control and one for the fertilized trees. The dependent variable used was BA increment during 1970-75 and as independent variable was used the initial BA. The results of this analysis (Tables 6,7Appendix 3) showed that in both cases there was a close relationship between BA increment and initial BA. The regression coefficient of the fertilized trees was greater than that of the control ones indicating the improvement in the increment induced by fertilization. The smaller correlation coefficient in the case of fertilized trees might be attributed to the variability within the site which was confounded by fertilization and/or probably to inherited genetic characteristics of the individual trees in the stand.

SUMMARY

Summarizing the foregoing analyses the following conclusions may by drawn:

1. BA was increased following fertilization and the increase started at the year that fertilizers were applied.
2. The maximum of the response occurred at the second year since fertilizers were applied, and in the fifth year the response was below that for the first year.
3. Dominant trees responded more than co-dominants and the latter more than the subdominants.
4. The response in BA increment was statistically significant at $p < 0.01$.
5. From the statistical analysis of the results is indicated that in cases where different treatments are compared, the increment during the under examination period is more sensitive variable to be examined than cumulated variables (e.g. BA in 1975), at the end of the period.

6. Another way that helped to unmask the response of BA at the end of the under examination period was analysis of covariance using as a covariate the initial BA of the trees.

7. When the response was examined at the end of the examination period it was found be only 11.5% of the control mean BA in 1975, but when BA increment was examined for the 1970–75 period the response was estimated as 56% of the corresponding mean control BA increment.

8. Finally, examination area increment at half of total height revealed statistically significant differences ($p < 0.01$) between control and fertilized trees and therefore pointing to the direction that trees were responding to the treatment not only at the breast height position but also higher up the stem.

6.3 HEIGHT RESPONSES RELATED TO FERTILIZATION

After the examination of the responses of BA and cross-sectional area at half total height, the next variable to be examined was the total height of the trees. Total height was defined in chapter 2 as the distance between the top and the position where a tree was cut.

There is evidence from other fertilization experiments that height responds to fertilization, but in some cases it does not. Brix (1976) reported responses in height in a thinning-fertilization (N) experiment of a 24-year old Douglas fir stand. Brix and Ebell (1969) reported also responses in height in a 20-year old Douglas fir stand following N fertilization. Heiberg, Madgwick and Leaf (1964) reported responses in height growth following K fertilization of a 30 to 35-year old red pine plantation. Woolons and Will (1975) reported no responses of height in a N fertilization experiment of naturally regenerated 13-year old radiate pine stand. As far as Sitka spruce is concerned, responses of height have been reported following PK fertilization of stands in pre-canopy closure stage of development (MacIntosh, 1978). Also responses in height of 5-year old Sitka spruce have been reported by Farrell and McAleese (1972) following NP fertilization. In this study all the trees sampled had been measured for total height as well as for annual height increment (Chapter 3). After that it was easy to follow the height development of these trees. In the next Table the situation prevailing at the beginning of the experiment (1970) is presented for the control and fertilized trees.

TABLE 11

STATISTICS OF TOTAL HEIGHT AT THE BEGINNING OF THE EXPERIMENT (1970)

Number of trees (n)	Treatment	Mean Height (m)	Standard Deviation	Variance	Standard Error	Range
40	Control	12.00	1.44	2.08	0.228	4.90
40	Fertilized	11.99	1.43	2.05	0.227	6.30

From the entries of the above table it appears that both control and fertilized trees had almost equal mean total height initially.

EXAMINATION OF THE FERTILIZATION EFFECTS IN TOTAL HEIGHT (TH)

Five years later, following fertilization the situation changed and is presented in the next table.

TABLE 12

STATISTICS OF TOTAL HEIGHT AT THE END OF THE EXAMINED PERIOD (1975)

Number of trees (n)	Treatment	Mean Height (m)	Standard Deviation	Variance	Standard Error	Range
40	Control	14.01	1.62	2.64	0.257	6.00
40	Fertilized	14.86	1.48	2.21	0.235	5.90

The entries of the above table indicated that fertilized trees achieved better growth and therefore giving a response of 2.87-2.01 = 0.86m in terms of mean height increment during the five year period, or almost 43% of the control increment during 1970-75. In order to examine whether these differences might be attributed to fertilization analysis of variance was applied to the data. The results of this analysis (Table 8 , Appendix 3) indicated that there were statistically significant differences between treatments as well as between blocks. At this stage a decision was made to remove variation in height by using analysis of covariance and using as covariate the initial height of the trees (TH in 1970). This time the results (Table 9 , Appendix 3) indicated highly significant differences ($p<0.01$), between control and fertilized trees. Therefore fertilization had a significant effect in terms of total height at the end of the examined period (1975).

EFFECT OF DOMINANCE CLASS

The situation terms of TH increment during 1970-75 in the three dominance classes of the control and fertilized trees is presented in the Tables 13 and 14 below.

TABLE 13

MEAN HEIGHT INCREMENT OF THE THREE DOMINANCE CLASSES DURING 1970-75 (CONTROL TREES)

Year	Suppr.-Subdominant Class (n=12) Total Height (m)	Co-dominant Class (n=14) Total Height(m)	Dominant Class (n=14) Total Height (m)
1975	12.80	13.89	15.20
1970	11.00	11.96	12.69
Mean Height Increment	1.80	1.93	2.51

TABLE 14

MEAN HEIGHT INCREMENT OF THE THREE DOMINANCE CLASSES DURING 1970-75
(FERTILIZED TREES)

Year	Suppr.-Subdominant Class (n=12)	Co-dominant Class (n=11)	Dominant Class (n=17)
	Total Height (m)	Total Height (m)	Total Height (m)
1975	13.40	15.32	15.54
1970	10.85	12.37	12.55
Mean Height Increment	2.55	2.95	2.99

Comparison of the control and fertilized height increment in the above two Tables revealed that fertilized trees had bigger height increment than the control ones. Comparison of height increment between the three dominance classes per treatment indicated that:

1. In the case of the control trees the dominant ones had the biggest height increment with the suppr.-subdominants having the smallest, while the co-dominant ones achieved an intermediate height increment.
2. In the case of the fertilized trees the situation was the same but with the co-dominant trees showing similar increment to the dominants.

The analysis of height increment response - defined as height increment of the fertilized minus the height increment of the controls - during the same period is presented in the next Table.

TABLE 15

HEIGHT INCREMENT RESPONSE TO FERTILIZATION DURING 1970-75
OF THE THREE DOMINANCE CLASSES

Treatment	Suppr.-Subdominant Class Increment m	Co-dominant Class Increment m	Dominant Class Increment m
Fertilized	2.55	2.96	2.99
Control	1.80	1.93	2.51
Response	0.75	1.02	0.48

This time the results indicated that co-dominant trees responded better to fertilization and also that dominant trees showed the smallest response.

HEIGHT INCREMENT AND RESPONSE DEVELOPMENT DURING 1970-75

Height increment during 1965-75 is presented in Fig. 22 . In this figure it can be seen that both groups of trees had the same trend in total height development during 1965-70. Following fertilization in 1970 the two height increment lines assume different trends. In Fig. 23 is presented the height increment development of the three dominance classes during 1970-75. In all cases the lines representing the fertilized classes during 1971-72 - the second year following fertilization - assume different trends. Finally in Fig.24 is presented the development of mean height response during the first five years following fertilization. In this figure it is indicated that fertilized trees achieved the biggest response during the second year following fertilization. After that year the response started declining and finally in the fifth year reached a level below that for the first year.

EXAMINATION OF THE RELATIONSHIP BETWEEN HEIGHT IN 1970 AND HEIGHT IN 1975

Examination of the relationship between the heights at the beginning and at the end of the under examination period using regression analysis was applied to the data. In both cases (control and fertilized) the correlation coefficient was high (0.94; Tables 10 and 11 , Appendix 3), and therefore proving the closeness of this relationship.

DISCUSSION

Height response is probably more complex than diameter response. We have seen in Figs. 22 and 23 that height development of the fertilized trees during the year of treatment was very slightly apart from that of the control (the height development lines are almost parallel). From the

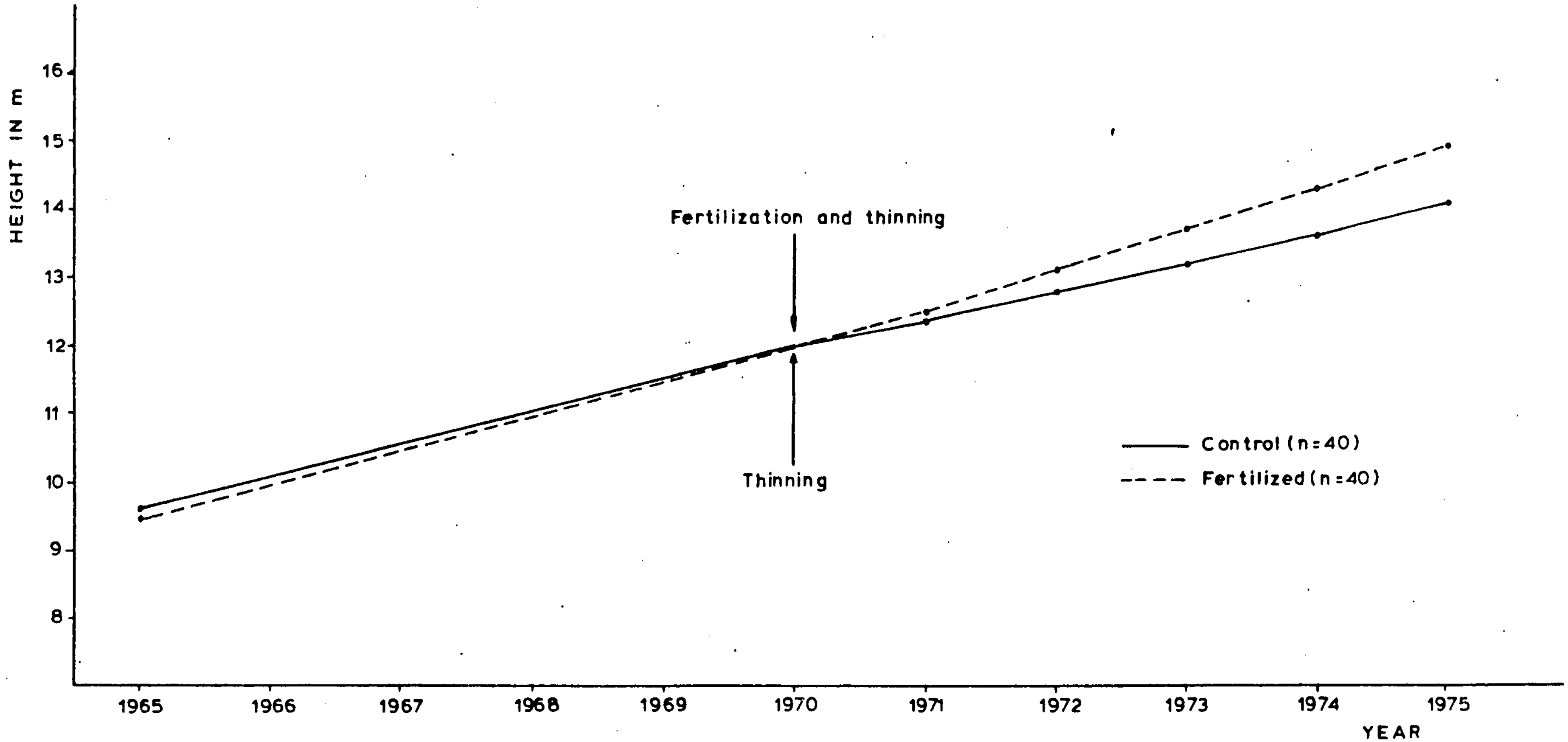


Fig. 22 Mean height development following fertilization.

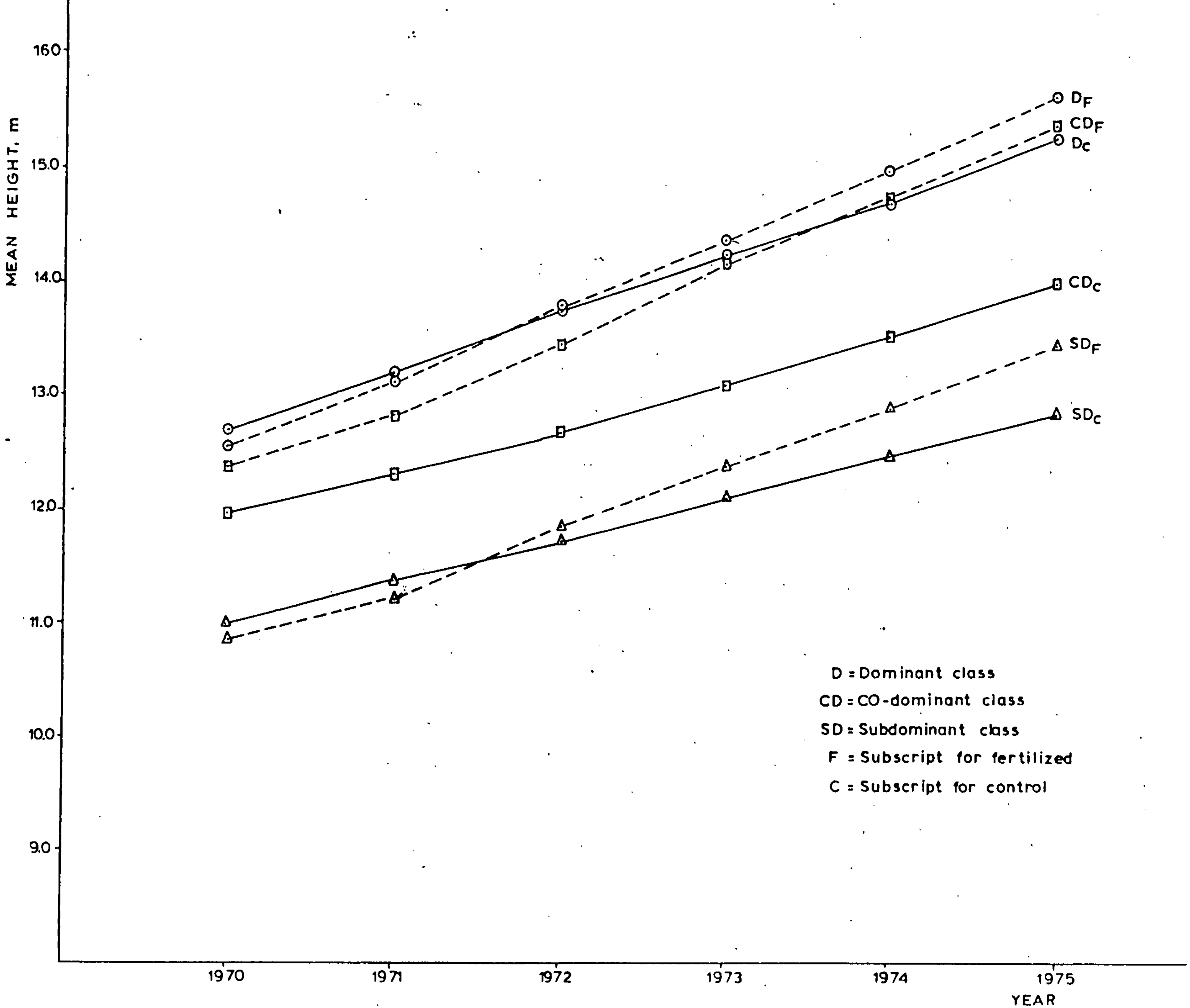


Fig.23 Mean height development of the three dominance classes during 1970-75 control and fertilized trees

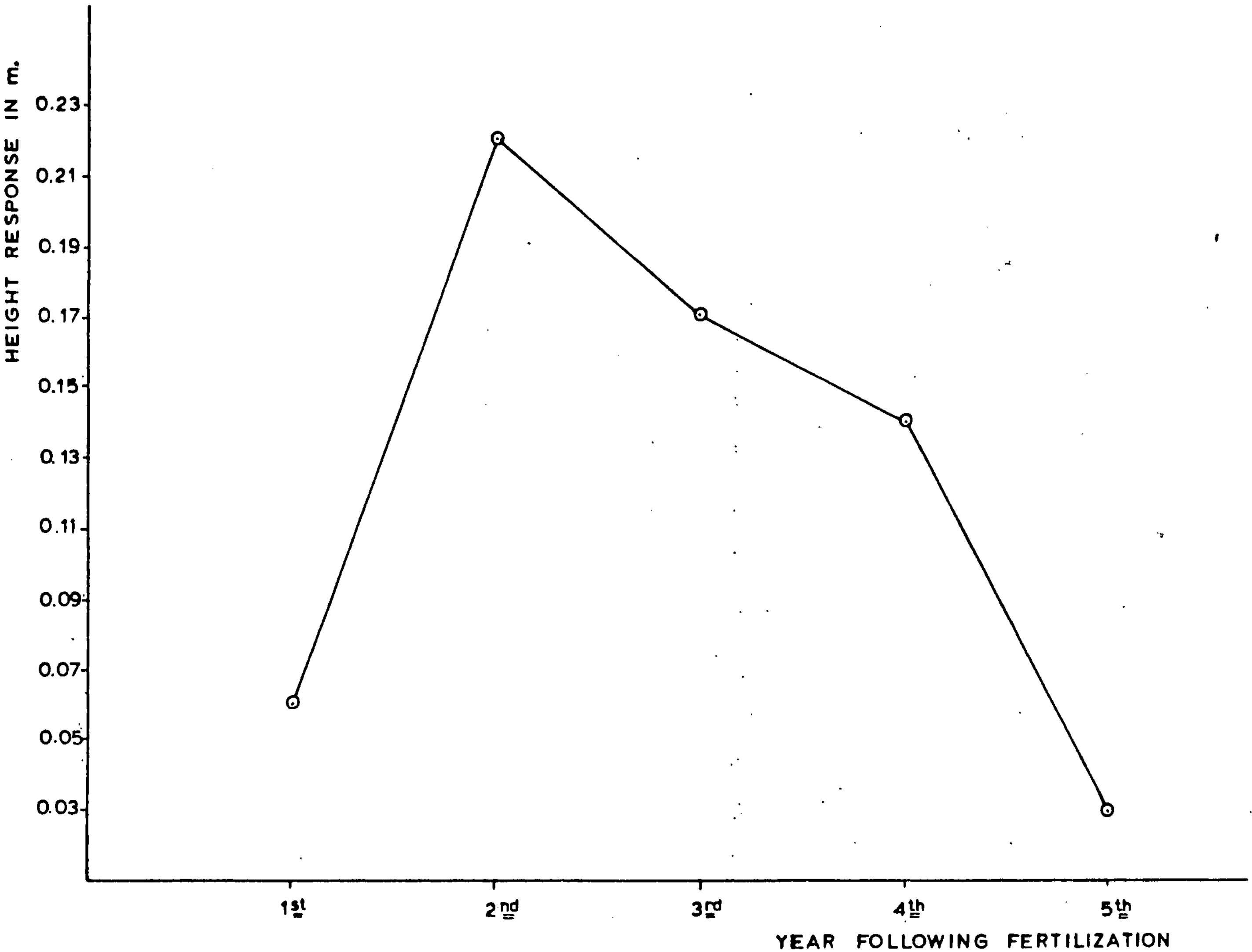


Fig. 24 Mean annual height response

second year onwards the response is characteristic (Fig. 23). This delay in height response is probably due to the fact that height growth must be divided and examined into two years because in each of these years influence could be exerted on a different phase of height growth.

Height development of a tree is formed in two phases (Duff and Nolan, 1953). The first phase takes place in one year during which the terminal bud is formed, and the second phase occurs in the following year when the bud elongates. Both phases are influenced by external conditions. Therefore during the year of fertilization the favourable conditions induced by the treatment added to formation of better organ primordia in the terminal bud which probably contributed to the better response in the following year (second since fertilization). Similar responses to fertilization with the maximum of the response taking place at the second year since the treatment have been reported by Brix and Ebell (1969). The pattern of the development of the height response was similar with that of basal area, that is both maximized at the second year and after that both started declining, but the per cent increase in response of basal area was higher than that of height (56% for basal area increment and 43% for height increment) indicating that probably height responses were not as large as basal area responses.

Another interesting feature in the analysis of height responses was that of co-dominant trees responding more to treatment than the dominant ones and the latter less than the subdominant ones. This was quite opposite with the responses of basal area. Similar responses of height to fertilization have been reported by Miller and Cooper (1973).

SUMMARY

Summarizing the responses of height to fertilization the following are worth noting:

1. The height of trees increased following fertilization.

2. The maximum of the response occurred in the second year following fertilization.
3. After the second year the response started declining and in the fifth year reached a level below that of the first.
4. Trees of the co-dominant class responded more to the treatment than the ones of the dominant class. The trees of the supr.-subdominant class showed an intermediate response.
5. The differences in height between control and fertilized trees in the end of the under examination period were statistically significant ($p < 0.001$).
6. The pattern of the development of the height response was similar with that of basal area but smaller in per cent terms.
7. Regression analysis showed a very close relationship between the heights at the beginning of the experiment (1970), and the heights at the end of the five year period (1975).

6.4 FORM FACTOR RESPONSES RELATED TO FERTILIZATION

The form factor of a tree is the amount by which the volume of a cylinder of identical basal area and height has to be multiplied to give the actual volume of a tree. Therefore form factor is calculated by the formula (13) below:

$$F = \frac{V}{G \times H} \qquad (13)$$

Where F is the form factor, V is the actual volume of the tree, G is the basal area and H is the total height of the tree.

There have been previous reports of changes in F following the application of fertilizers (Woolons and Will, 1975; Whyte and Mead, 1977). There have been also reports of F remaining unaltered with slight changes in the taper of the trees (Miller and Cooper, 1973).

Finally, Mitchell and Kellog (1972) suggested that volume responses to fertilization in dominant trees from breast height measurements should possibly be approached with caution. So far there have not been any reports about changes in Sitka spruce, following the application of fertilizers.

In this study the volume of the trees sampled was estimated as it is described in paragraph 6.5 Having estimated the volumes of the trees as well as having their basal areas and heights (para. 3.6, 3.3) it was easy to estimate the form factors by applying formula (13). Form factor was estimated for all trees, for each year from 1970–75. In the next table are presented statistics of FF in 1970, the year that fertilisers were applied.

TABLE 16

STATISTICS OF FORM FACTOR AT THE BEGINNING OF THE EXPERIMENT (1970)

Number of trees	Treatment	Mean Form factor	Standard Deviation	Variance	Standard Error	Range
40	Control	0.480	0.0397	0.00157	0.0063	0.193
40	Fertilized	0.468	0.0393	0.00154	0.0062	0.169

The above results indicate that the mean form factor of the fertilized trees was lower than the corresponding of the control trees at the year of treatment. The difference was 0.480–0.468 = 0.012 or 2.5% of the mean control form factor for that year.

Five years following fertilization the situation changed into the one presented in Table 17.

TABLE 17

STATISTICS OF FORM FACTOR AT THE END OF THE UNDER EXAMINATION PERIOD (1975)

Number of trees	Treatment	Mean Form factor	Standard Deviation	Variance	Standard Error	Range
40	Control	0.483	0.0348	0.00121	0.00550	0.163
40	Fertilized	0.484	0.0353	0.00125	0.00558	0.122

The entries of the above table indicate that during the five year period the mean form factor of the fertilized trees reached the corresponding of the control trees, which during the same period remained almost unaltered.

EXAMINATION OF FORM FACTOR CHANGE DEVELOPMENT DURING 1970–75

The development of form factor change in both groups of trees during the under examination period is presented in Figs. 25 and 26. In the first figure in order to have a connection with the previous situation in the stand, five years before fertilization, the mean form factors for 1965 were estimated and added. In Fig.26 it can be seen that the mean form factor of the fertilized trees started increasing in the same year following the application of fertilizers (1970). This increase continued up to the fourth year. Finally during 1975 the increase dropped, so that both groups of trees had the same mean form factor. During the same period the form factor of the control trees underwent only minor changes. This change in the form factor of the fertilized trees could be considered as the result of the changes in the basal area, height and volume of the trees.

We have already seen that at the beginning of the experiment there was a difference of 2.5% between the mean form factors of the control and fertilized trees. We have also seen that in 1975 – five years following fertilization – both groups of trees had the same mean form factor. Because of that it was decided to test the change in form factor for the control and fertilized trees for the period 1970–75. For this purpose analysis of variance was applied. The result of this analysis (Table 12, Appendix 3) revealed that there was a statistically significant difference at $p < 0.01$. In order to examine if there were similar trends in form factor change during the five year period before fertilization (i.e. 1965–1970), form factors were estimated for both tree groups in

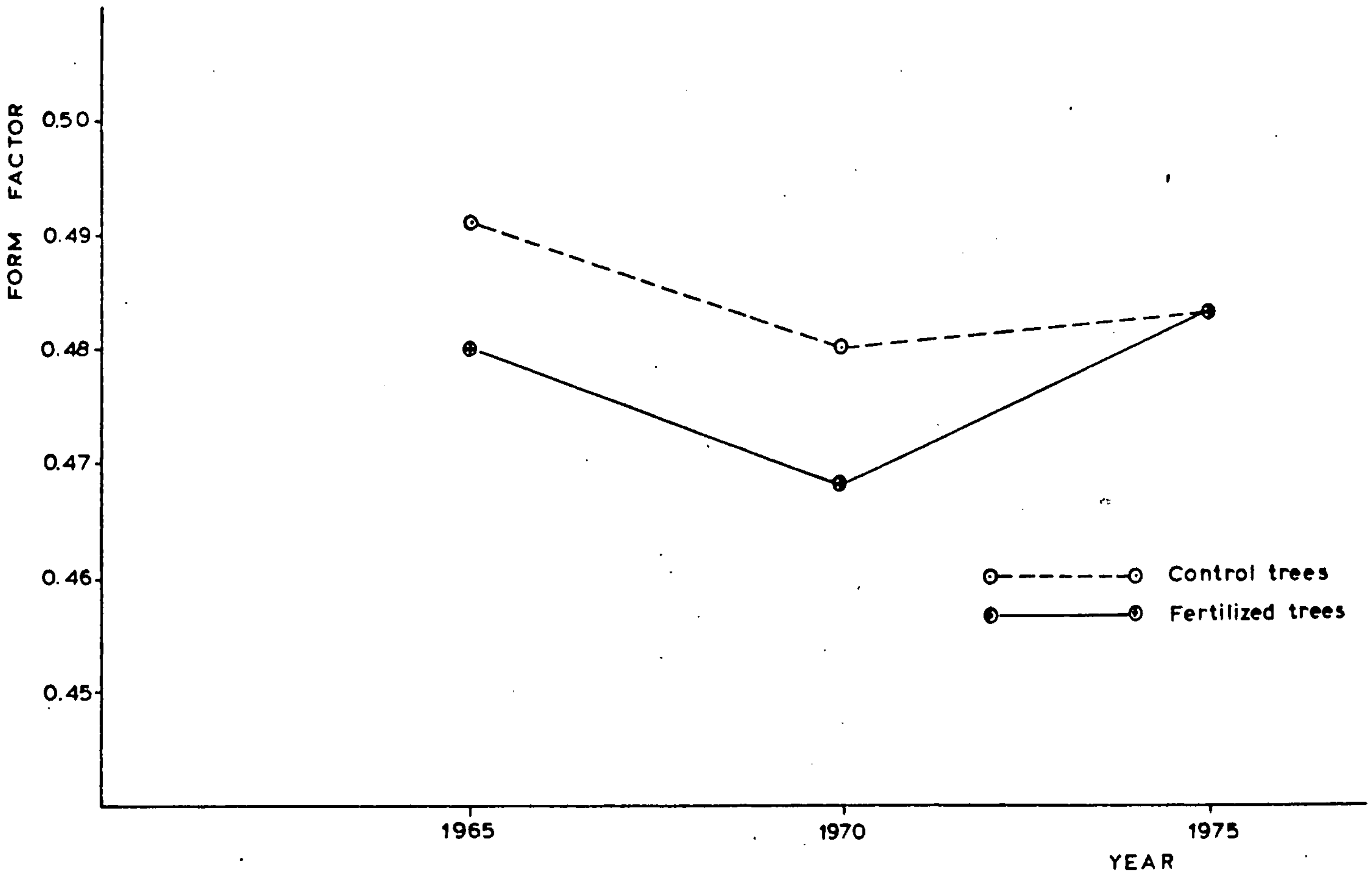


Fig.25 Mean form factor response

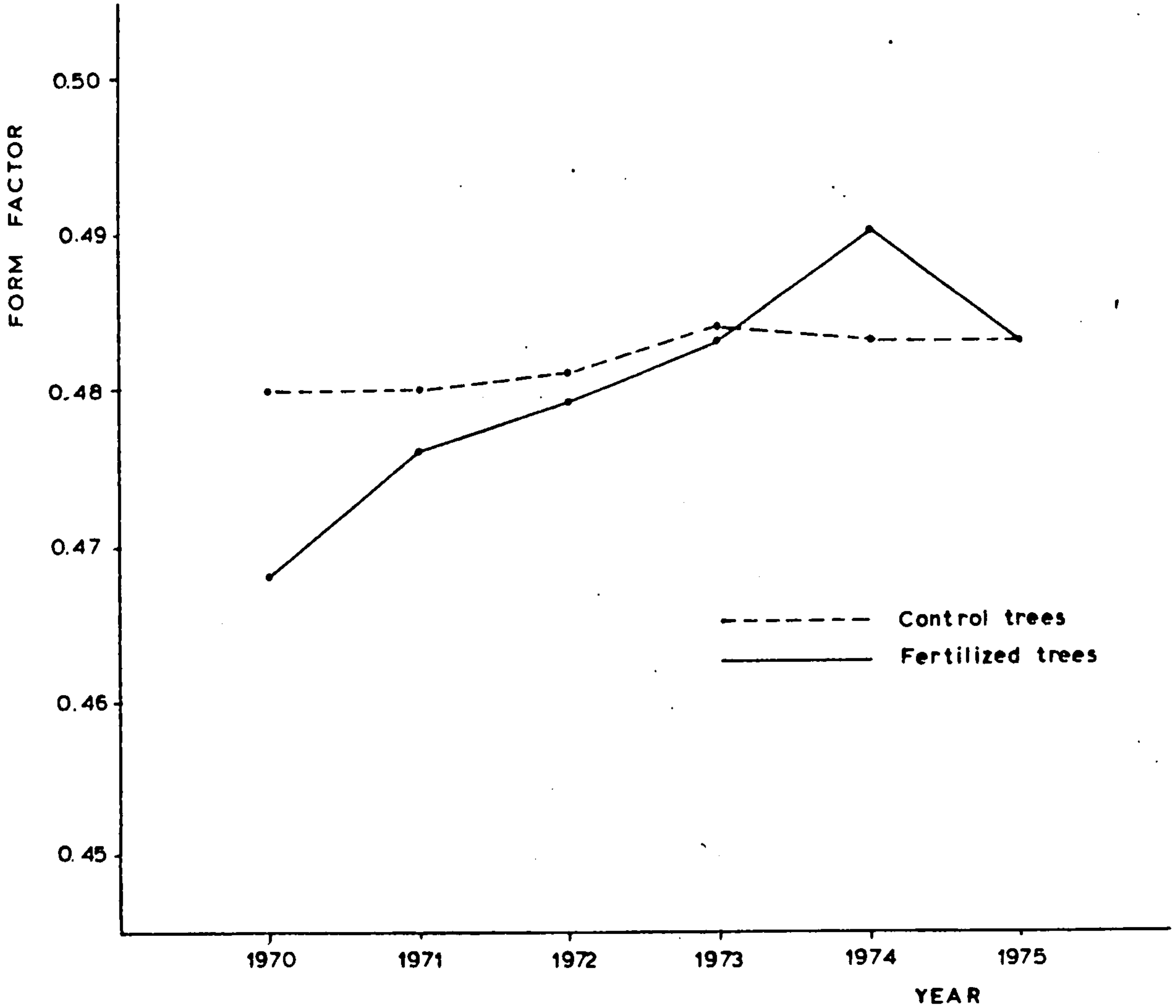


Fig.26 Mean form factor development during 1970-75

1965 and the change in form factors was derived easy since form factors for 1970 had already been estimated. Analysis of variance was applied again and the results (Table 13 , Appendix 3) indicated that there were no differences in form factor change during 1965–70 for the control and fertilized trees respectively, and therefore adding to the previous fact that the change in form factor would be attributed to fertilization.

EFFECT OF DOMINANCE CLASS

In the next two tables as well as in Figs.27 and 28 is presented the development of form factor change in the three dominance classes.

TABLE 18

MEAN FORM FACTOR CHANGE OF THE THREE DOMINANCE CLASSES DURING 1970–75 (CONTROL TREES)

Year	Supr.-Subdominant Class (n=12)	Co-dominant Class (n=14)	Dominant Class (n=14)
1975	0.490	0.493	0.466
1970	0.491	0.488	0.463
Form Factor change	– 0.001	0.005	0.003

TABLE 19

MEAN FORM FACTOR CHANGE OF THE THREE DOMINANCE CLASSES DURING 1970–75 (FERTILIZED TREES)

Year	Supr.-Subdominant Class (n=12)	Co-dominant Class (n=11)	Dominant Class (n=17)
1975	0.491	0.479	0.481
1970	0.488	0.471	0.452
Form Factor change	0.003	0.008	0.029

The entries of the above tables as well as the figures indicate the following:

1. Control trees: In 1970 the trees of the dominant class had

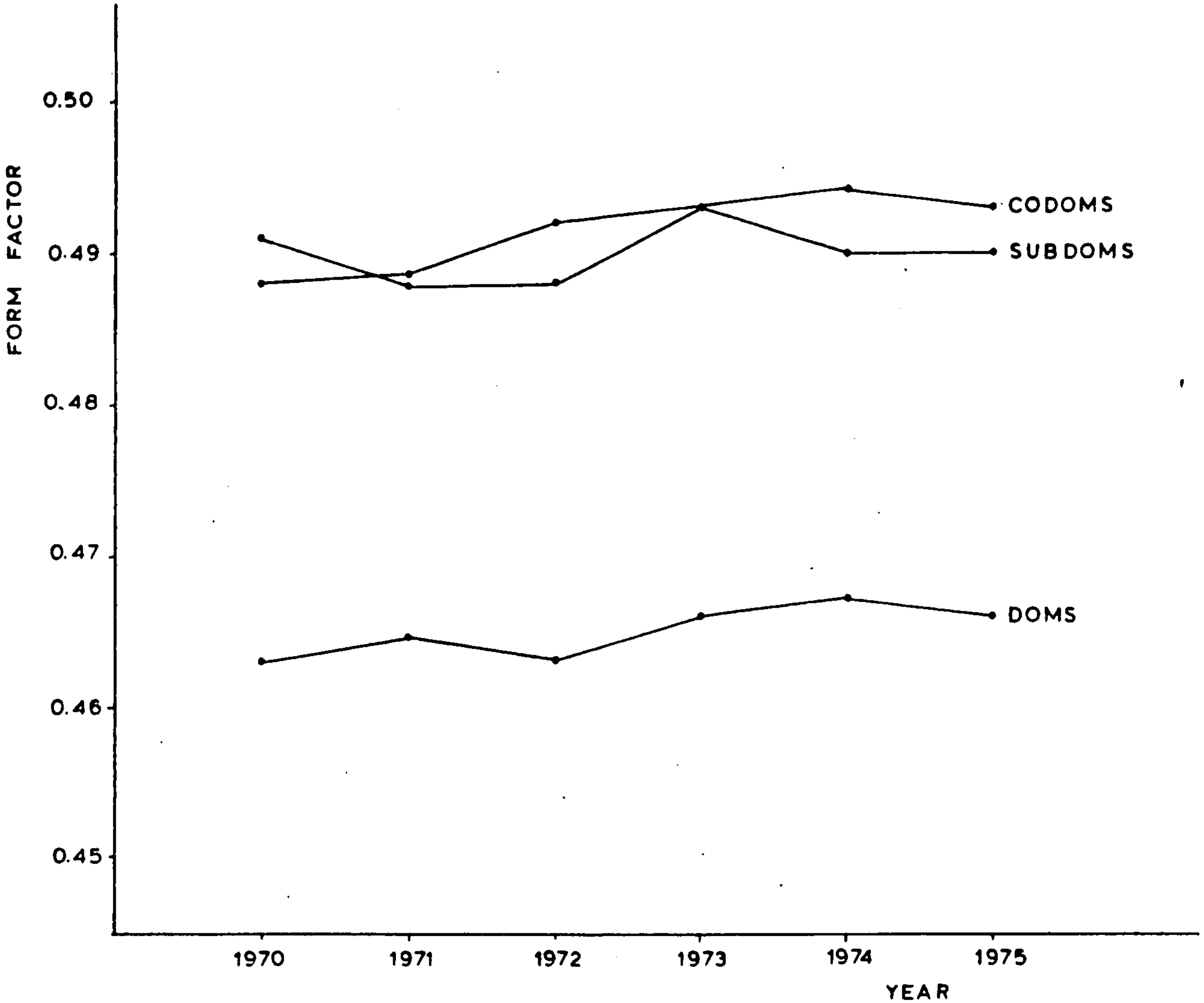


Fig.27 Mean form factor development of the three dominance classes, during 1970-75, of the control trees.

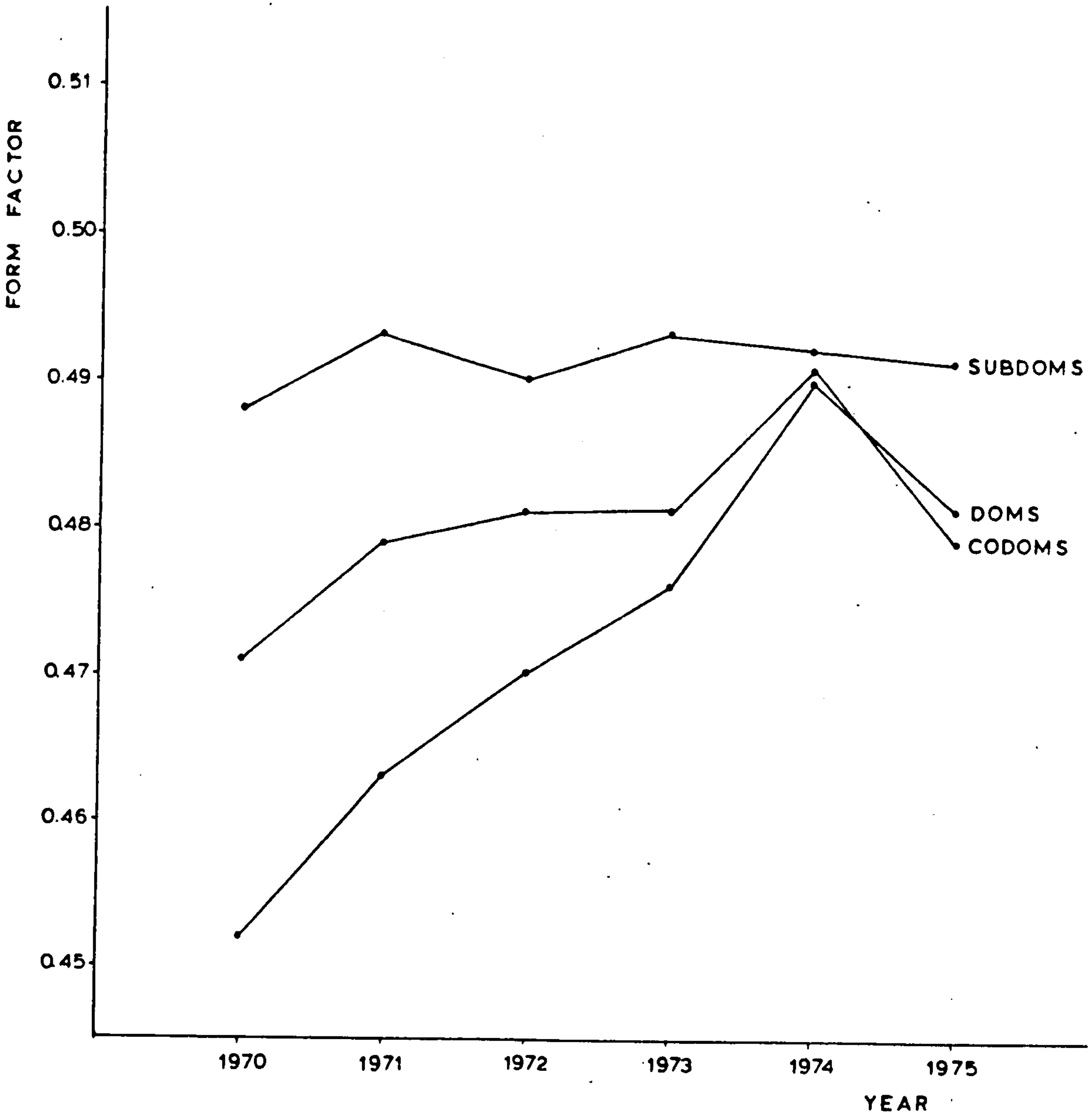


Fig. 28 Mean form factor development of the three dominance classes, during 1970-75, of the fertilized trees

the smaller form factor, the co-dominant ones were in the middle and the supr.-subdominants had the biggest. In 1975 the situation was the same though trees of the co-dominant class were very slightly improving their form factor. The changes of all classes were of minor importance.

2. Fertilized trees: In 1970 the situation was the same as in the case of the control trees with the difference that form factors of all classes were smaller than the corresponding of the control trees. In 1975 trees of the subdominant class still had the biggest form factor, the trees of the co-dominant class had the smallest, while the dominant trees had a form factor slightly bigger than that of the co-dominants. Comparison of the form factor change between the three classes during 1970-75 indicated that dominant trees underwent the biggest change - 0.029 - as compared with the two other classes (0.008 and 0.003 for the co-dominant and supr.-subdominant trees respectively.

Finally the response of form factor defined as the difference in change between control and fertilized form factors during 1970-75, indicated that the fertilized dominant trees responded more to the treatment. The trees of the two other classes showed minor responses only (Table 20).

TABLE 20

FORM FACTOR RESPONSES OF THE THREE DOMINANCE CLASSES DURING 1970-75

Treatment	Supr.-Subdominant Class change	Co-Dominant Class change	Dominant Class change
Fertilized	0.003	0.008	0.029
Control	- 0.001	0.005	0.003
Response	0.004	0.003	0.026

SUMMARY

Summarizing the form factor responses to fertilization the following could be concluded:

1. Fertilization increased the mean form factor of the trees.
2. Examination of the response of the form factor change during the under examination period between control and fertilized trees revealed that there were statistically significant differences ($p < 0.01$).
3. Examination of the response of the mean form factor change among the three dominance classes of the fertilized trees indicated that dominant trees were the ones responding most to the treatment.
4. The mean form factor of the control trees remained almost unaltered during the under examination period.
5. The response to fertilization started the year that fertilizers were applied and dropped in the fifth year.
6. The pattern of the response development of form factor changes was the result of the combined effects of basal area, height and volume of the trees.

6.5 VOLUME RESPONSES RELATED TO FERTILIZATION

So far the responses of basal area,height and form factor to fertilization have been examined. Since these three variables are closely related with the volume of the trees, it might be expected that there would be corresponding changes in the volume of the trees.

There have been reports of volume responses to fertilization of Douglas fir (Bower 1973), radiata pine (Woolons and Will, 1975), black spruce (Weetman, 1975), white spruce (Gagnon *et al*, 1976) and Loblolly pine (Matziris and Zobel, 1976).

VOLUME ESTIMATIONOF THE SAMPLED TREES

In this study the volume of each of the trees sampled was estimated using the sectional method. This involved estimation of the volume of each section by applying the formula for the frustrum of the conoid (Chapter 4), while the volume of the top part for each tree was estimated using the formula for a cone. Finally summing up the volumes of the individual sections, the volume of each tree was estimated. Since the diameters were measured from discs cut at each mid-internodal position it was easy to estimate the volume of each of the trees for each subsequent year, either before or after fertilization (stem analysis).

6.5.1
VOLUME RESPONSE OF THE SAMPLED TREES

In the next Table are presented the statistics of volume of the control and fertilized trees in 1970, at the beginning of the experiment.

TABLE 21

STATISTICS OF VOLUME AT THE BEGINNING OF THE EXPERIMENT (1970)

Number of trees	Treatment	Year	Total volume (m^3)	Mean volume (m^3)	Standard deviation	Variance	Standard error	Range
40	Control	1970	3.299	0.082	0.037	0.001	0.006	0.154
40	Fertilized	1970	3.257	0.081	0.032	0.001	0.006	0.117

The entries of the above Table indicate that at the beginning of the experiment both groups of trees had almost equal mean volumes.

EXAMINATION OF THE OVERALL EFFECTS IN VOLUME

Five years later (following fertilization in 1970) the situation changed (Table 22), and there was a difference in mean fertilized tree volume of 0.022 $\pm$ 0.026 or 18.2% over the corresponding mean control tree volume in 1975.

TABLE 22

STATISTICS OF VOLUME AT THE END OF THE PERIOD (1975)

Number of trees	Treatment	Year	Total volume (m^3)	Mean volume (m^3)	Standard deviation	Variance	Standard error	Range
40	Control	1975	4.837	0.121	0.057	0.003	0.009	0.223
40	Fertilized	1975	5.726	0.143	0.059	0.004	0.009	0.259
Response (Fert.-Cont.)			0.889	0.022				

In order to examine this difference between the volumes of the control and fertilized trees in 1975, analysis of variance was used (Table 14, Appendix 3). The results of the analysis did not show any statistically significant differences between treated and untreated trees, but they indicated differences between blocks probably because of the site variability in the experimental area. Because of that it was decided to use analysis of covariance using as covariate the volume of the trees in 1970. The results (Table 15 , Appendix 3) indicated statistically significant differences between treated and untreated trees at $p < 0.01$, a result that would be attributed to fertilization.

EXAMINATION OF VOLUME INCREMENT DURING 1970-75

Examination of the volume increment during 1975 (Tables 21 and 22) indicated that the mean control increment was $0.039m^3$ and the mean fertilized increment was $0.062m^3$, and therefore giving a response of $0.023 \pm 0.012m^3$, or nearly 59% of the corresponding mean control increment during the same period. Examination of the significance of the response of volume increment was carried out using analysis of variance. This time the results of the analysis (Table 16 , Appendix 3) showed statistically significant differences between treatments at $p < 0.01$.

EFFECT OF DOMINANCE CLASS

In the next Tables is presented the volume increment as well as its response during the period 1970-75.

TABLE 23

MEAN VOLUME INCREMENT OF THE THREE DOMINANCE CLASSES DURING 1970-75 (CONTROL TREES)

	Supr.-Subdominant Class (n=12) m3	Co-dominant Class (n=14)m3	Dominant Class (n=14)m3
Volume 1975	0.0750	0.1078	0.1734
Volume 1970	0.0550	0.0741	0.1144
Volume increment	0.0200	0.0337	0.0590

TABLE 24

MEAN VOLUME INCREMENT OF THE THREE DOMINANCE CLASSES DURING 1970-75 (FERTILIZED TREES)

	Supr.-Subdominant Class (n=12) m3	Co-dominant Class (n=11)m3	Dominant Class (n=17)m3
Volume 1975	0.0830	0.1461	0.1862
Volume 1970	0.0514	0.0860	0.0997
Volume increment	0.0316	0.0561	0.0865

TABLE 25

MEAN VOLUME INCREMENT RESPONSE OF THE THREE DOMINANCE CLASSES DURING 1970-75

Dominance Class	Supr.-Subdominant class	Co-dominant class	Dominant class
Response	0.0316 - 0.0200 = 0.0116	0.0561 - 0.0337 = 0.0224	0.0865 - 0.0590 = 0.0275

The results of the above Tables indicated that fertilized dominant trees responded more to treatment than trees of the two other classes despite the fact that in 1970 they had smaller mean volume than the corresponding mean of the control dominant class. Co-dominant trees showed also good response to the treatment and finally, trees of the suppressed-subdominant class showed the smallest response. The same results are presented also in Fig. 29.

VOLUME INCREMENT AND RESPONSE DEVELOPMENT DURING 1970-75

Mean annual volume development as well as mean annual volume increment development of both treated and untreated trees during 1970-75 is presented in Figs. 30 and 31 respectively. It is interesting to note in Fig.31 that mean annual volume increment of the fertilized trees was increasing from the first and up to the fourth year, and after that drops (5th year). During the same period mean annual volume increment of the control trees follows the same pattern and during the fifth year levels off. In the same figure it is also characteristic the difference in rate of growth between treated and untreated trees, the former having higher than the latter.

Finally in Fig. 32 the development of mean volume increment response, defined as mean fertilized volume increment minus mean control volume increment, is presented. In this figure it is indicated that volume increment response started the year that fertilizers were applied and reached its maximum during the fourth year since fertilization. After that and during the fifth year dropped and reached a level below the corresponding response of the first year. The drop of the response in the third year might be attributed to the beneficial effect of thinning in the increment of the control trees which reached a maximum in the third year and therefore reducing the response.

USE OF REGRESSION ANALYSIS FOR THE EXAMINATION OF DIFFERENT RELATIONSHIPS AS AFFECTED BY FERTILIZATION

Several regressions were developed to examine the relationships between volume or volume increment with basal area, and also to examine

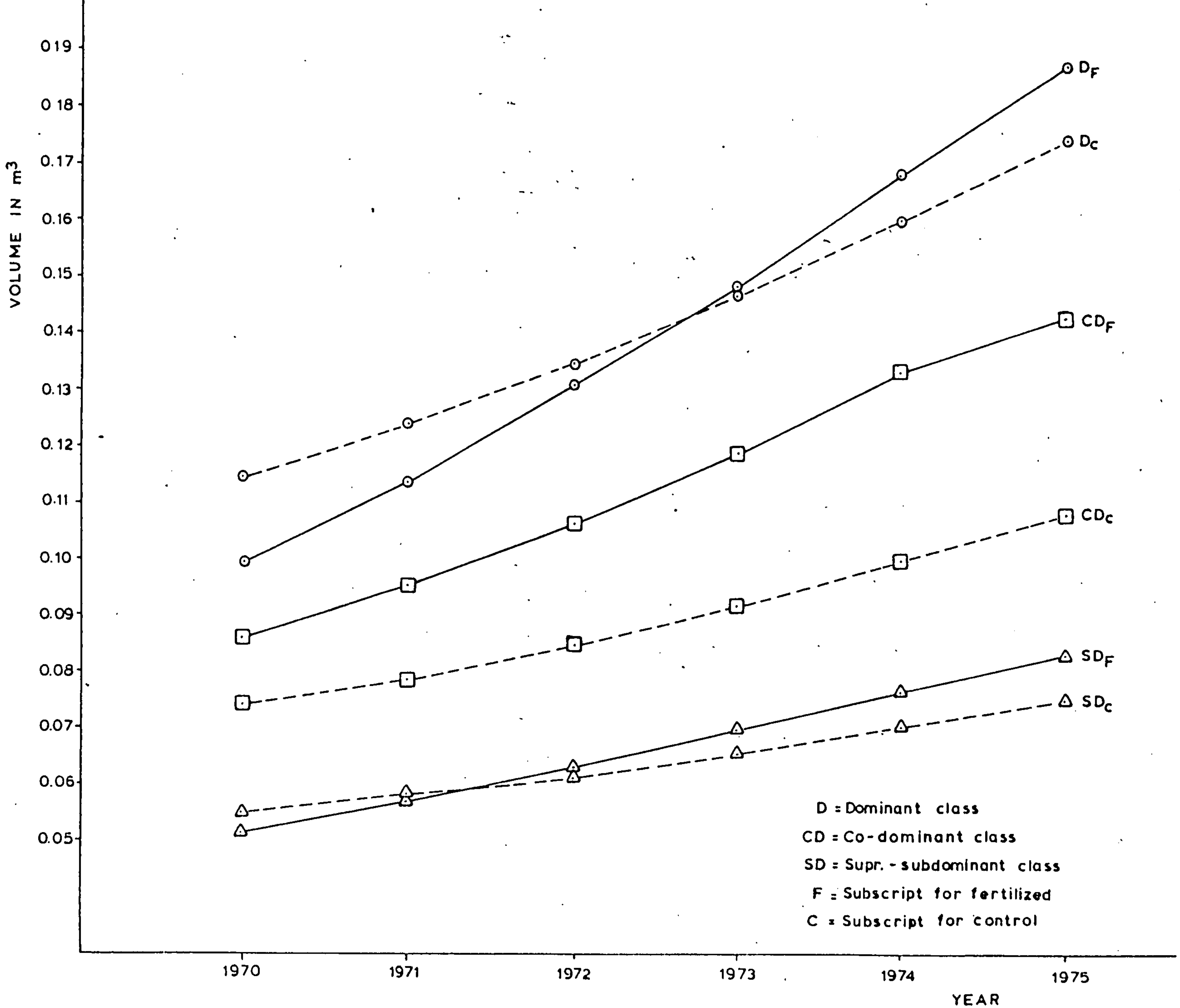


Fig.29 Mean volume development of the three dominance classes of the control and fertilized trees

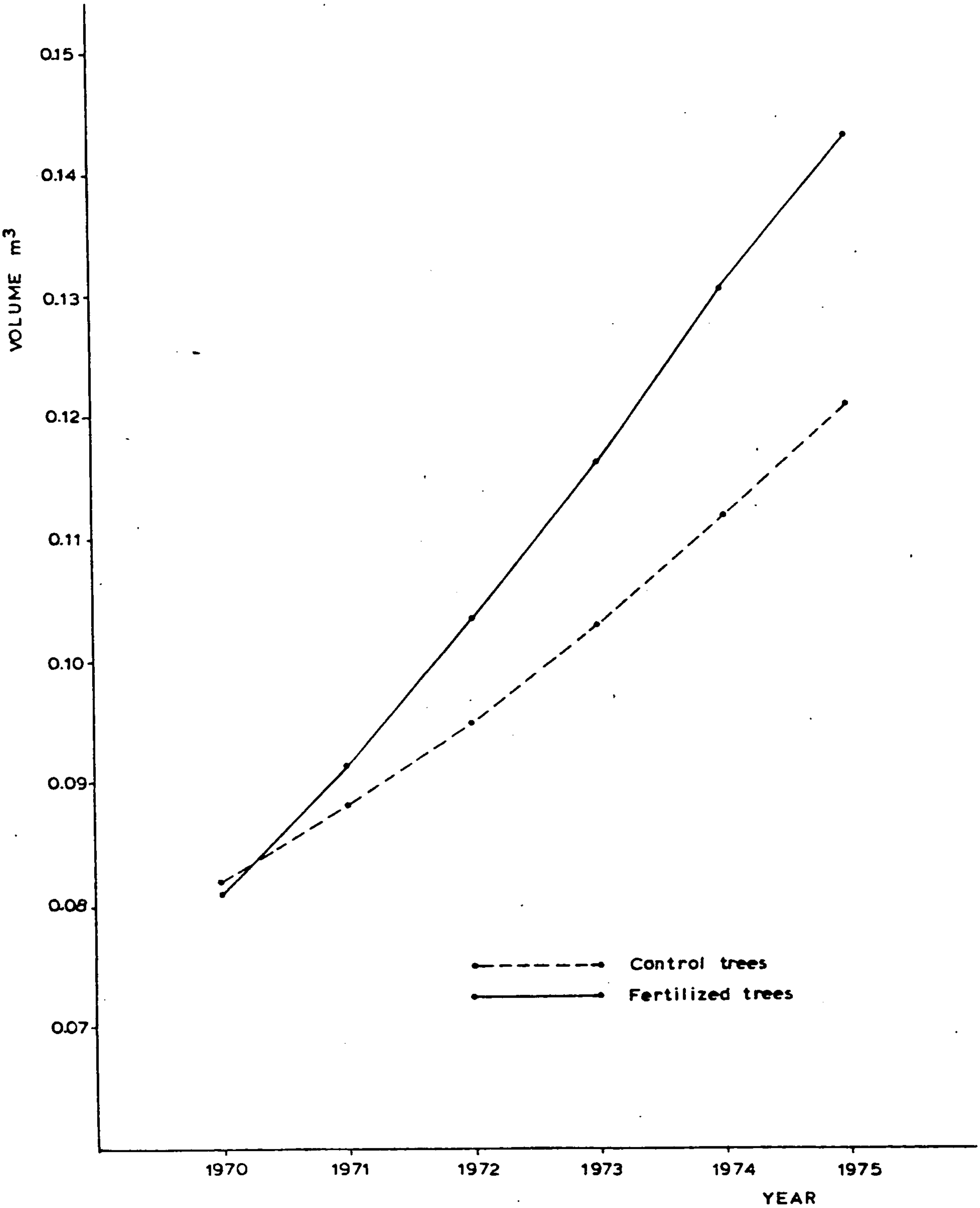


Fig.30 Mean annual volume development during 1970-75

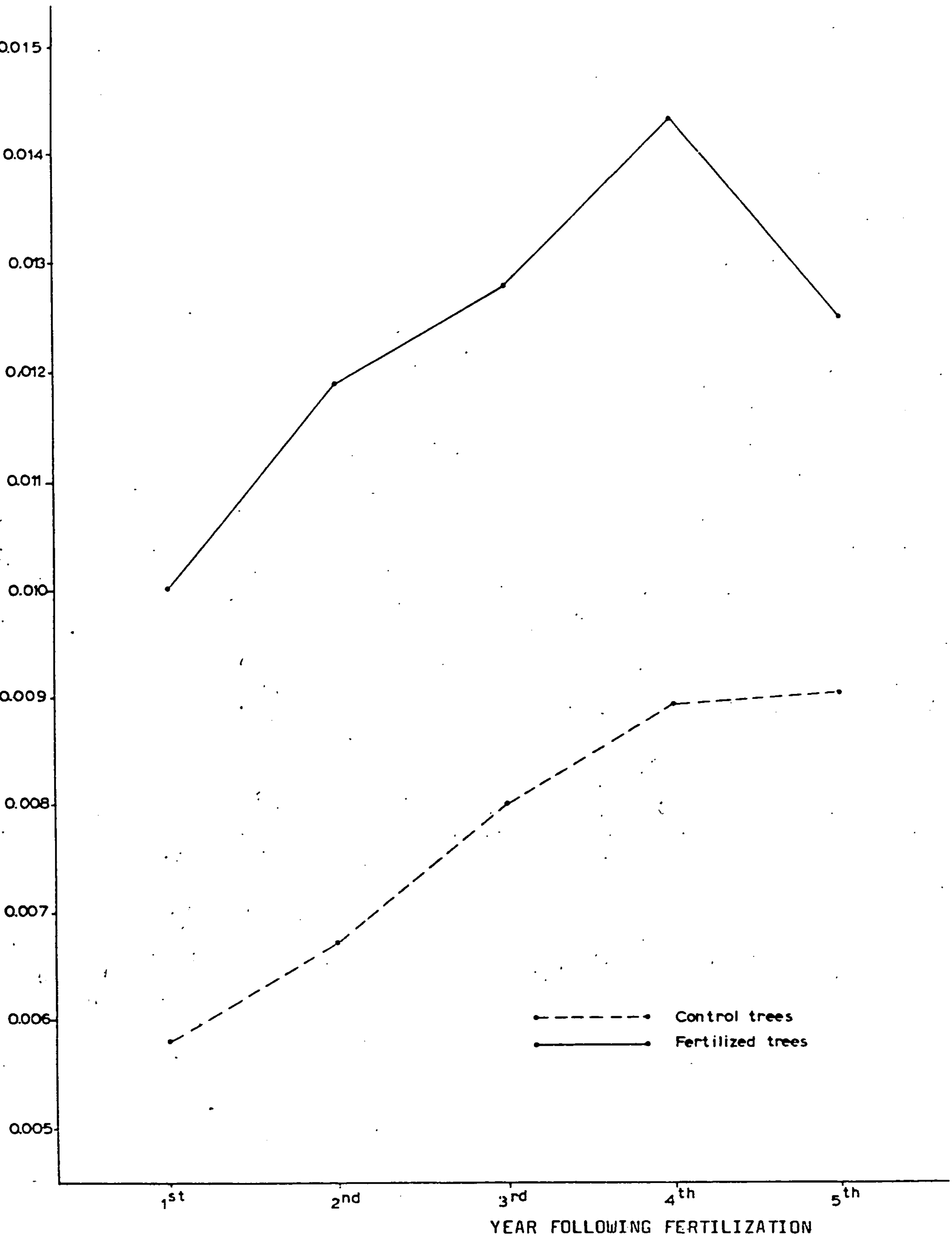


Fig.31 Mean annual volume increment development

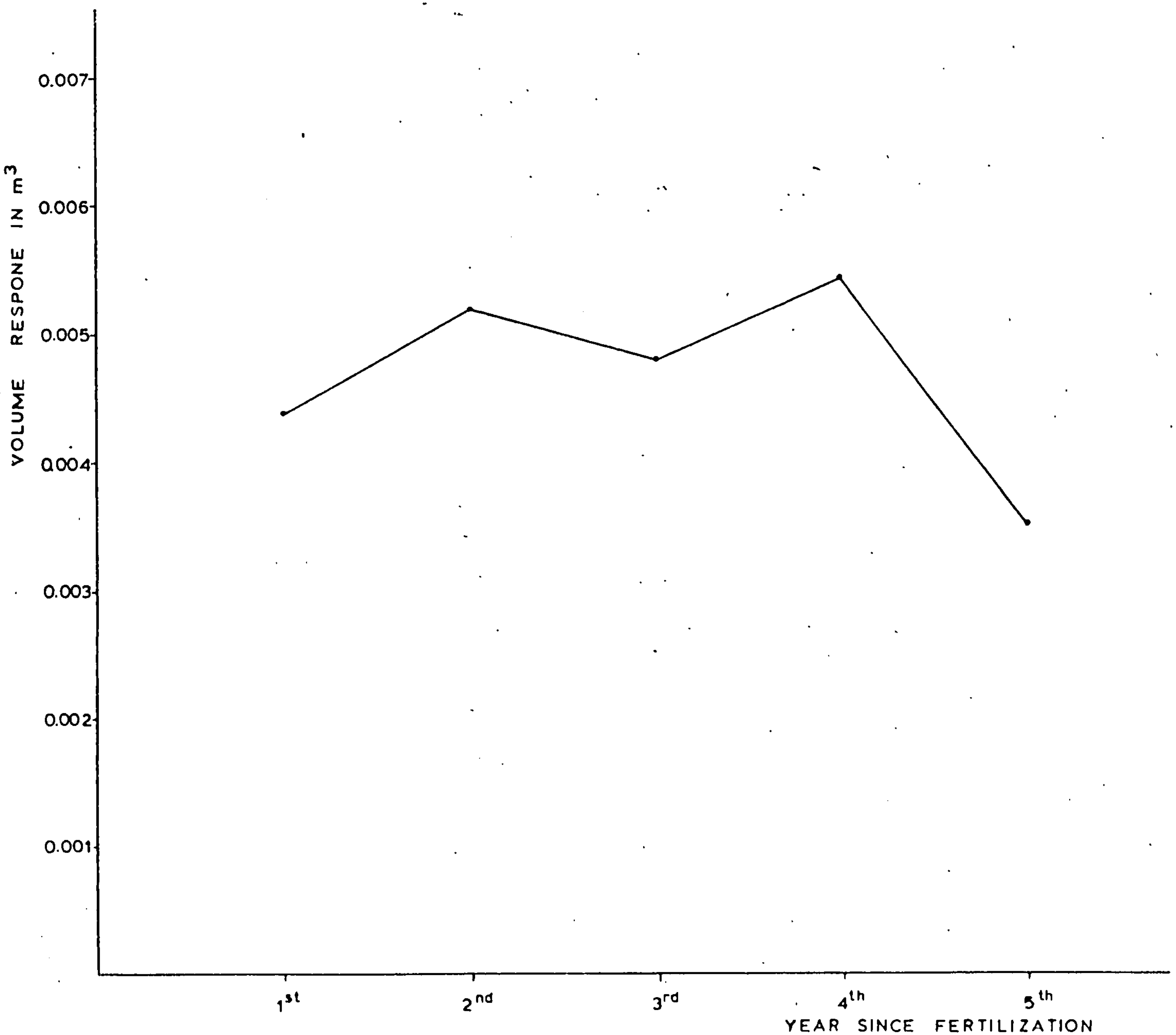


Fig.32 Mean volume increment response development

the relationship between initial (1970) and final volumes of the control and fertilized trees.

An overall regression of volume increment during 1970-75 with basal area in 1970 (Table 17, Appendix 3) indicated a systematic overestimation in the predicted values of the control trees volume increment, and at the same time the volume increment of the fertilized trees was underestimated (Fig. 33). This result indicated the need to use separate regressions for the control and fertilized trees in the case that volume increment was going to be estimated by means of basal area at the beginning of the experiment. Two such regressions were developed one for the control and one for the fertilized trees (Table 18a , and 18b, Appendix 3) with correlation coefficients 0.895 and 0.831 respectively. Comparison of these regressions (Table 19 , Appendix 3) indicated that there were statistically significant differences in both coefficients and constants ($p < 0.01$), and therefore justifying the indications of Fig. 33 and proving the need to use two separate regressions in order to estimate volume increment from basal area measurements taken at the beginning of the experiment, in case fertilization has an effect in the increment of the trees.

Next it was decided to examine the relationship between initial (1970) and final volumes (1975) for the control and fertilized trees. Two regressions were developed for this purpose and a very good relationship was found in both cases ($R = 0.986$ and 0.950 for the control and fertilized trees respectively, Table 20 , Appendix 3). In order to see if only one regression could be used for both groups of trees (treated and untreated) comparison of the two regressions was carried out. The results (Table 21 , Appendix 3) revealed that there were statistically significant differences ($p < 0.01$) in both coefficients and constants, proving again the need for two separate regressions to be used in cases of estimating fertilization responses.

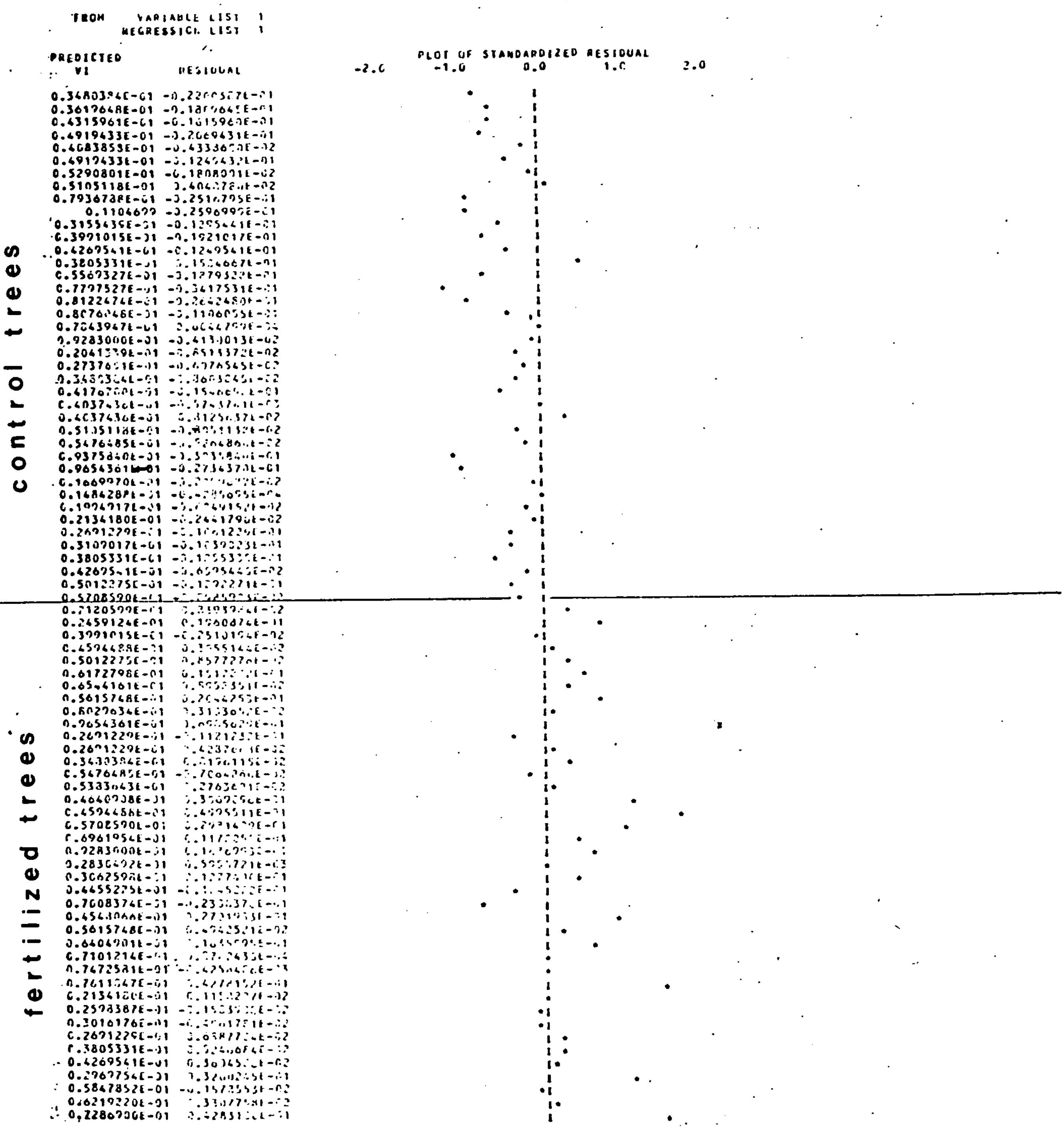


Fig. 33. Plot of the standardized residual from the regression of volume increment 1970-75, with BA 1970.

Finally, the relationship between volume and basal area in 1975 was examined for both groups of trees. Two such regressions were developed and the results (Table 22, Appendix 3) indicated that in both cases the relationship was very good (R=0.967 and 0.971 for the control and fertilized trees respectively). Comparison of the two regressions in this case revealed that there were no differences (Table 23, App.3) between constants as well as no differences between coefficients, and therefore pointing to the direction that one regression could be used for both control and fertilized trees. This result was contrary to the previous regression comparisons where statistically significant differences were found between treated and untreated trees. An indirect conclusion that might be drawn in this case is that increment is a more sensitive variable to be used in cases of estimating fertilization responses.

SUMMARY

Summarizing the results of volume responses to fertilization the following are worth noting:

1. Fertilization increased the volume of the trees.
2. The response of volume of the mean tree defined as the difference in the mean tree volumes in 1975 was $0.022m^3$, or 18.2% over the corresponding mean control tree volume in 1975.
3. The response of mean tree volume increment defined as the difference in mean tree volume increment between control and fertilized trees was $0.023m^3$, or 59% over the corresponding control mean tree volume increment during 1970-75.
4. Statistical analysis of the differences in 1975 volume between control and fertilized trees did not show statistically significant differences when analysis of

variance was applied, but statistically significant differences were revealed ($p < 0.01$), when analysis of covariance was used (covariate volume in 1970).

5. Analysis of variance of volume increment showed statistically significant differences ($p < 0.01$) between treated and untreated trees.
6. Examination of the volume response of the three dominance classes indicated that trees responded to fertilization in accordance with their position in the stand canopy.
7. Examination of the development of the volume response indicated that it started in the first year, reached its maximum in the fourth and finally in the fifth year dropped below that of the first year.
8. Close relationships were found between:

   a) the volumes of the trees at the end of the under examination period with the corresponding volumes at the beginning of the period, and therefore suggesting that trees responded in accordance with their initial volumes. Comparison of the regressions of volume 1975 with volume 1970 between control and fertilized trees showed statistically significant differences ($p < 0.01$) for both coefficients and constants.

   b) volume increment and basal area at the beginning of the experiment. Overall regression of volume increment with initial basal area indicated the need to use two separate regressions, one for the control and one for the fertilized trees. Comparison of these two regressions revealed that there were statistically significant differences between coefficients and constants ($p < 0.01$), and therefore proving the need for two separate regressions to be used for purposes of the estimation of response in terms of volume increment.

c) volume in 1975 and basal area in 1975 for both groups of trees (treated and untreated). Comparison of the two regressions did not show any statistically significant differences ($p < 0.05$).

6.5.2
ESTIMATION OF VOLUME RESPONSE ON AN AREA BASIS

After the examination of volume response of the 80 sampled trees, it was decided to estimate the response per hectare. Unfortunately complete enumeration of DBH of the trees in the experimental area was not carried out during 1970, but such data were available for 1971. For this reason the following analysis of volume response estimation per hectare was restricted to the period 1971-75.

From the volume data estimated from the sample trees, it was easy to develop regression equations to expand sample tree volumes to crop volumes. It is well known that there is a very close relationship between volume, basal area and height. Multiple regressions relating volume with basal area and height give very good volume estimates, but present certain practical difficulties since the total height of every tree in the crop has to be measured accurately. When height is estimated from a sample and expanded to the rest of the population by means of an easily measured independent variable, one will have to face other sources of errors (i.e. sampling errors, regression errors) which will be added to the general volume model. On the other hand, there is a close relationship between volume and basal area (volume-basal area line, Hummel, 1955) which is more simple to use and quite accurate in volume estimates. Models relating volume or volume increment to basal area were developed and used in the methods for the volume response estimation of this study. Trees with DBH < 7.5cm were not included in the following analysis since they were considered as contributing very slightly to the volume responses.

METHOD 1

In this method it was decided to use direct volume estimates at the end of the period under examination (1975) for the control and fertilized trees respectively, and examine their differences as an estimate of the response. For this purpose two regressions of the form:

$$V = a + b\ G$$

were developed, one for the control and one for the fertilized trees.

The data used for the development of these regressions were the volumes (v) of the 40 control and 40 fertilized trees, which were estimated from stem analysis for 1975, and the basal areas (G) of the trees which were estimated from the overbark diameters measured in the forest in 1975. Details about these regressions are given in Table 26 below and more information is given in Table 24 in Appendix 3.

TABLE 26

REGRESSION OF VOLUME 1975 (UNDERBARK) WITH B.A. 1975 (OVERBARK)

Treatment	Number of trees	a	b	R	Reggressions F
Control	40	-0.0295	7.5102	0.970	605.018**
Fertilized	40	-0.0390	8.0843	0.975	751.036**

** statistically significant at $p < 0.01$

Tree volumes were estimated using regression estimators of the form

$$V_i = a + b\ G_i$$

using the appropriate a and b for each treatment as well as the basal area G of each tree. In the next table are presented the volumes of the trees of all the plots per treatment as they were estimated from the above regressions.

TABLE 27

VOLUMES OF ALL THE TREES OF THE FOUR PLOTS PER TREATMENT ESTIMATED BY REGRESSIONS OF THE FORM $V_{75} = a+b\ G_{75}$

Treatment	Number of trees (all plots)	Total volume m3	Mean volume m3	Standard error (mean volume tree)
Control	240	18.123	0.0755	0.00267
Fertilized	254	21.041	0.0828	0.00280
Response		2.918	0.0073	

The results of the above table showed that the mean tree response in 1975 was 0.0073 ± 0.0078 m^3 and for the total number of the trees of the four plots/treatment (240 and 254 control and fertilized respectively) the response was estimated as 2.918 ± 1.879 m3.

It has already been seen in Chapter 2 that there were four plots/treatment each of them an area of 0.02ha. Therefore the above estimated response was over an area of 4 x 0.02 = 0.08 ha. By expanding the above results on a per hectare basis (Freese, 1962),that is for an average of 3,000 control and 3,150 fertilized trees/ha respectively, the estimated volume response/ha with 95% confidence limits was 34.247 ± 23.389 m^3/ha. Comparison of the two regressions showed that there were neither statistically significant differences between the coefficients nor between the constants (Table 25 , Appendix 3).

METHOD 2

In this method the response was estimated as the difference in volume increment, during 1971-75, between control and fertilized trees. For this purpose two regressions of the form:

$$VI = a + b\ G$$

were developed, one for the control and one for the fertilized trees. The data used in these regressions were volume increment (VI) underbark from stem analysis during 1971-75 as the dependent variable, and as independent variable was used the basal area (G) in 1971 overbark, both estimated from the sampled trees. Details about the regressions are given in Table 28 below, and more information is given in Table 26 in Appendix 3.

TABLE 28

REGRESSION OF VOLUME INCREMENT (UNDERBARK) WITH BASAL AREA 1971 (OVERBARK)

Treatment	Number of trees	a	b	R	Regressions F
Control	40	-0.0113	2.618	0.887	141.70**
Fertilized	40	-0.0234	4.309	0.885	113.39**

**statistically significant at $p < 0.01$

Volume increment during 1971-75 of all the trees in the four plots per treatment were estimated using the same formula as in the previous method. The results are presented in Table 29 below.

TABLE 29

VOLUME INCREMENT DURING 1971-75 OF ALL THE TREES OF THE FOUR PLOTS PER TREATMENT ESTIMATED BY REGRESSIONS OF THE FORM $VI = a+b\ G_{71}$

Treatment	Number of trees	Total Increment m^3	Mean Increment m^3	Standard error
Control	240	4.824	0.0201	0.00151
Fertilized	254	7.468	0.0294	0.00262
Response		2.644	0.0093	

The results of the above table showed that the volume increment response with 95% confidence limits for the mean tree was $0.0093 \pm 0.0061 m^3$. For the total number of trees/treatment (240 control and 254 fertilized) covering an area of $0.02 \times 4 = 0.08$/ha, the estimated response was $2.644 \pm 1.487\ m^3/0.08$ ha. By expanding the above results on a per hectare basis the response was estimated (with 95% confidence limits) as

$32.30 \pm 18.48\ m^3/ha$.

Comparison of the results of this method with the previous one indicates that this method gives slightly higher precision (since the standard error is smaller).

Finally comparison of the two regressions of this method developed for the control and fertilized trees indicated that there were statistically significant differences in both coefficients and constants at $p < 0.01$ (Table 27, Appendix 3) and therefore proving the necessity for using two separate regressions, one for the treated and one for the untreated trees.

DISCUSSION OF THE RESULTS OF THE TWO METHODS

We have already seen in the previous paragraph that the second method based on volume increment gave a better estimation of the response of the mean tree - $0.0093\ m^3$ as compared with $0.0073\ m^3$ of the first method. Comparison of the mean tree volume responses estimated using these two methods with the corresponding mean tree volume responses, as estimated from the sample of 40 single trees, reveals that there are big differences. In the case of the single trees, the mean tree volume responses were estimated as $0.022 m^3$ and $0.023 m^3$ respectively (para. 6.5.1.) Of course the response of volume in the case of single trees was estimated for the period 1970-75, that is one year more than the corresponding period 1971-75 used for the volume estimation based on the regressions developed for the two methods, but still the differences in response are considerable. One of the reasons might be attributed to the sampling procedure that was followed, since more weight was given to the bigger trees to be included in the sample (chapter 2, para. 2.3), and therefore not being representative of the situation prevailing in the experimental plots. Another reason that might be considered is that all the regressions developed for the volume estimation on a unit area basis used basal area as the independent variable.

This meant that these volume functions still ignore any changes in form factor that might have occurred as a result of fertilization, and in this study we have already seen that there was a 2.5% change in form factor. Reukema (1971) reported that "the error in estimated 4-year growth associated with ignoring a 1 per cent change in form factor was generally 5 to 10 per cent, or even greater, depending on tree size and growth rate" in an experiment to estimate the volume growth of Douglas Fir. Taking this into consideration would bring the volume responses on a unit area basis closer to the responses of the single trees.

6.6 SUMMARY AND DISCUSSION OF THE RESULTS

Summarizing the effects of fertilization on basal area and height there are several points worth noting.

Both these two variables showed the same general pattern of response (Figs.21 and24). Basal area responded the year that fertilizers were applied while the response of height during that year was very small (Figs. 22 and 23). This small response of height during the first year was probably partly attributed to the thinning carried out in the experimental area which reduced the height increment, a fact that has been observed for various tree species and referred to as a "shock" effect (Staebler 1956, Miller 1961, Berry 1969, Brix 1976). This might also be attributed to the fact that height growth depends upon a two year period and it was during the second that the influence of fertilization, induced during 1970 (first year), contributed to the maximum increase in the second year since fertilization (Heiberg <u>et al</u>, 1964). On the other hand, the increase in basal area during the first year might be explained as a combined effect of mechanical trends developed at the lower part of the tree (Assmann, 1970) as a result of thinning, and/or because of the increased photosynthetic area (Brix, 1976) of the trees as a result of the fertilizers which were applied early and before the initiation of the growth period in 1970. Both basal area and height reached the maximum response during the second year following fertilization, and after that started declining. The difference was that the response of height increment was relatively smaller than that of basal area increment (42.8% as compared to 56%). This result in turn might influence the taper of the trees and/or their form factor. Statistical analysis of the responses of basal area and height were statistically significant.

Examination of BA responses in dominance classes revealed that trees of the higher canopy were responding more tothe treatment. In the case of height responses it was the co-dominant tree class that was responding more to the treatment (Miller and Cooper, 1973).

Estimation of the form factors and analysis of the responses revealed that the form factors of the dominant fertilized trees changed during the five year period, while the form factor of the control trees remained almost unaltered. This change was statistically significant as compared with the corresponding change of the control trees. Examination of the form factor development during 1970-75 revealed that it was increased more during the first and the fourth year, a result that was very similar with the volume development during the same period. This seemed reasonable since form factor $F = \frac{V}{G \times H}$ is directly proportional to volume and the increases in height were small at the first and fourth years. Mitchell and Kellog (1972) suggested that the estimation of volume responses to fertilization in dominant trees "should be approached with caution". Johann, 1977, and Sterba, 1976, reported changes in form following fertilization, and Whyte and Mead (1977) reported changes in form factor of radiata pine.

Such changes in form factor as a result of a redistribution of increment on the tree stem caused by fertilization can be important for the forest managers since it will influence the distribution of wood between different log lengths.

Finally, examination of the volume revealed - as it was expected from the changes in BA and height - that fertilization increased the volume increment of the trees. The responses of volume and volume increment were relatively higher than those of BA (18.2% and 59% as compared with 11.5% and 56% respectively), probably reflecting the change in the form factor of the trees.

In all the cases of examining the differences between treated and untreated trees, increment - that is the difference in a variable at the beginning and the end of a period - was more "sensitive" in unmasking the response caused by the treatment.

In all the cases examined, trees of the higher canopy (dominant trees) were associated with the greatest response to the treatment (apart from the case of height responses). Kozlowski and Peterson (1962) found that the length of the period of growth in individual trees was increased with dominance, and therefore they pointed that greater increment was associated with greater dominance. This was confirmed by Hamilton (1969) for Sitka spruce.

Examination of the volume response on a unit area basis revealed that the estimated mean tree response - in both methods used - was much lower than that estimated from the single trees. This was attributed to sampling errors as well as to the regressions used in which only the BA of the trees was taken into account and therefore any changes in form factor were ignored.

Finally examination of the duration of the response implicated that it lasted for almost four years and dropped in the fifth year since fertilization. This, as well as the other examined characteristics of the response, was in accordance with the results of the previous chapter, dealing with the examination of the internal pattern of growth of the trees.

CHAPTER 7

USE OF PRINCIPAL COMPONENT ANALYSIS TO DEFINE THE STEM FORM OF SITKA SPRUCE

7.1 INTRODUCTION

A knowledge of the form of tree stems and the way in which tree stems vary in form is of importance to the forester for the estimation of volume and the construction of volume Tables. Knowledge of form is considered necessary for comparison of stem form between species for morphological studies and also for proper evaluation of the results of silvicultural treatments such as thinning, pruning and fertilizing.

As noted in Chapter 4 it is frequently assumed that parts of the stem resemble strongly various geometric solids. Unfortunately deviations from these geometric forms are common. These deviations have been attributed to several causes such as butt-swell (Larson, 1963), silvicultural treatments (Myers, 1963), site (Burger, 1951), age (Bickerstaff, 1946), dominance (Horn, 1961) and heredity (Squillace and Silen, 1962). Finally Grosenbaugh (1966) pointed out that regardless of cause, tree stems were capable of assuming an infinite variety of shapes. The existance of an extensive literature dealing with the mathematical definition of the stem form, in spite of the aformentioned difficulties proves the importance of such a definition, as well as the difficulty of this task.

Because of the simplicity of assuming that taper of the central stem conforms to the dimensions of a geometric solid, foresters in the past have relied on a single curve for describing the stem form and estimating the volume of the trees (Metzger, 1893, Peterson, 1927, Tiren, 1928, Gray, 1956, Heger, 1965, Kozak , 1969). Use of one curve often failed to trace the inflection points of the stem curve and particularly those at the lower part of the stem (butt-swell) as well as within the crown of the tree. Max and Burkhart (1976) suggested that three polynomial models were needed for the description of the stem form, one for each segment of the bole (neiloid, paraboloid and conoid). They used a technique for joining these models into a single model which could be analyzed using regression (segmented polynomial regression). Preussner (1974), Heijbel (1974) and Jokela (1976) have also examined the problem of analysing the stem curve in different parts and using different functions for them. Demaerschalk and Kozak (1977) used two mathematical functions to describe the stem curve, which were joined at a point of inflection lying at a relative height 20 to 25% from the ground level.

As it might be expected, use of more than one curve improved the description of stem form but there were still inadequancies caused by the difficulty of determination of the position of the inflection points along the stem curve since it is very difficult to force living organisms - such as trees - subjected to the influence of a great number of environmental factors to present standard positions of inflection points. Therefore the difficulty of positioning the inflection points along the stem curve was still introducing bias even when more than one curve was used for the description of the stem form as in the aformentioned methods. Two more reasons could be mentioned here, in addition to the above, for the failure of polynomials or other mathematical expressions to trace the inflection points:

1. The fact that the measured diameters never agree exactly with a mathematical expression, and
2. because the recorded diameters present irregularities due to errors of observations and to actual irregularities

To overcome the problems of the great number of environmental factors influencing the development of stem form as well as the difficulties of tracing the inflection points of the stem curves Fries (1965) and Fries and Matern (1966) introduced multivariate analysis. Fries and Matern (1966) used principal component analysis to define the stem form of Betula verrucosa and Pinus silvestris from Swedish forest and compare them with corresponding species of British Columbia. Mendiboure (1972) used PCA to examine the influence of pruning on the stem form of young poplars. Finally Liu and Keister (1978) used this method to define the stem form of Pinus taeda and Pinus elliotii.

In this study among the other purposes were the examination of probable differences in stem form between control and fertilized trees as well as the development of a model for diameter prediction. Therefore in this Chapter in order to define the stem form of the Sitka spruce trees, carry out comparisons of the stem form and develop equations of diameter prediction, a multivariate method, Principal Component Analysis was used. The application of PCA in this study to define the stem form of the trees was based on the principles introduced first by Fries (1965) and Fries and Matern (1966) who examined the pattern of variation in stem form for different species.

7.2 PRINCIPAL COMPONENT ANALYSIS (PCA)

Principal component analysis is one of the subfields of "Multivariate analysis" which denotes analysis of data that are multivariate in the sense that each member bears the values of -p- variates. According to Anderson (1958) the multivariate character lies in the multiplicity of the -p- variates which are dependent among themselves. PCA is a technique intended to achieve an economical summarization of data sampled from a single population of multivariate normal measurements (Seal, 1964).

Statistical methods are usually applied to either test a hypothesis or to estimate a quantity in a hypothesis. Multivariate methods in most cases serve neither of these purposes but they are used to generate hypotheses, and since PCA is a multivariate method it is used for this purpose. Finally PCA can be described as an analytic procedure of orthological transformation from a set of correlated variates to another of uncorrelated variates.

Some of the objectives of PCA may include one or more of the following (Jeffers, 1978):

1. Examination of the correlations between the separate variables
2. Reduction of the basic dimensions of variability expressed by the individual sampling units to the smallest number of meaningfull dimensions
3. Elimination of the variables which contribute little extra information of the study

The basic theory of PCA is very well described by Kendall (1957), Anderson (1958), Seal (1964), and applications of this method in the field of forestry have been reported by Fries (1965), Fries and Matern (1966), Jeffers (1967), Mendiboure (1972), Liu and Keister (1978). In the application of PCA the following steps are distinguished (Jeffers, 1967):

1. Choice of the variables to be included in the analysis
2. Construction of the data matrix
3. Transformation of the basic data if required
4. Calculation of the dispersion or correlation matrix
5. Calculation of the eigenvalues and eigenvectors of the dispersion or correlation matrix
6. Examination and interpretation of the eigenvalues

7. Interpretation of the eigenvectors

8. Plotting or further analysis of the transformed values

The most important properties of the PCA (Seal, 1964, Kendall, 1957, Jeffers, 1978) can be described as follows:

Suppose we have -p- variates $x_1, \ldots x_p$, each observed on n individuals where x_{ij} is the j^{th} observation in the i^{th} variate. Since the object PCA is to economize in the number of variates we will seek for linear transformations of the type:

$$Z_i = \sum_{i=1}^{p} a_{ij} \; x_j \qquad (14)$$

In order to carry out an approximate reduction in the dimensions of the problem the coefficients -a- will be chosen such as that the first of the new variates, Z_1, has as large a variance as possible, the second variate, Z_2, will be chosen such as to be uncorrelated with the first and to have as large a variance as possible. The analysis then continues to find Z_3, Z_4 etc. until all the variability has been accounted for. The coefficients -a- of equation (14) can be considered as a vector of the form:

$$(a_{i1} \ldots\ldots a_{ip}) \qquad (15)$$

For each vector of the above form (15), corresponds a number -λ- which is one of the roots of the "eigen equation" (i.e. the equation that results after the mathematical manipulation of the correlation matrix). These vectors are called "eigen vectors" or "latent vectors". Each of the λ's is an estimate of the variance of the original variates after their linear transformation and is called "eigenvalue or "latent root". The important property of the estimated eigenvalues is that of the following equation:

$$\lambda_1 + \lambda_2 + \ldots \lambda_p = \sigma_1 + \sigma_2 + \ldots \sigma_p \qquad (16)$$

which means that the total variance of the new variates is equal to the total variance of the original variates. There are -p- vectors like (15) and each of them defines a set of independent variates which are called "Principal Components", $(Z_1, Z_2, Z_3 \ldots Z_p)$. In practice the estimation of $Z_1, Z_2, Z_3 \ldots Z_p$ is equivalent to extracting the eigenvalues and eigenvectors of the correlation matrix. An important property of this manipulation is that the linearily transformed set of new variates, $Z_1, Z_2 \ldots Z_p$ are mutually independent and can be considered separately (Seal, 1964).

The eigenvalue of the first principal component (Z_1), expressed as a percentage of the total for all variates indicates the proportion of the total variability accounted for by this component and the significance of the new variate as well. In the same way the remaining eigenvalues summarize the proportion of variability accounted by the appropriate component and those proportions can be added to give the cumulative proportions of variability accounted for by the linear functions. Because of the decreasing order of the variance associated with the principal components it is possible to use only one, two, or three of the new variates (principal components), to summarize the variability and covariability of the original variates, $x_1, x_2 \ldots x_p$ (Seal, 1964). Finally another property of PCA is that each of the original variates (x_i) can be expressed as a linear function of the eigenvectors, that is:

$$x_i = \sum_{i=1}^{p} a_{ij} Z_j \qquad (17) \text{ (Gittins, 1969)}$$

In geometrical terms (Gittins, 1969) the whole situation could be explained as follows:

Starting with the case of two variates (i.e. diameters) every individual can be represented by a point on a two axes plane. By extending the geometrical concept from the two axes plane into the N dimensional space (i.e. by including N variates) we have the entries of each row of the data matrix regarded as coordinates of points in the space and the position of each point will be determined uniquely by each particular combination of the observed values (positional diameters), and finally a cloud of points will be determined. The shape of this cloud of points will be the result of the relationships that exist betwenn them, and the closer the points the better the relationships between them.

The existance of strong correlations in the correlation matrices reflects the trend of the variables (positional diameters) to vary together and implies that the points in the cloud are contained in a restricted region of the space. This enables new coordinate axes to be strategically placed through the major dimensions of that particular region of space that the points actually lie. Hence each point will be assigned new values of coordinates corresponding to the new axes. The longer of the new axes will correspond to the direction of the maximum variability. These new axes are the principal components , and the longest direction of the elipse of the cloud of points will always correspond with the first principal component.

7.3. USE OF PCA TO DEFINE THE STEM FORM OF SITKA SPRUCE CONTROL AND FERTILIZED TREES

We have already seen in paragraph 7.1 that PCA was first used to define the stem form of the trees by Fries and Matern (1966). In their study they concluded that the first eigenvector gives the linear relationship between diameters and they considered this as the mean stem form of the trees examined in their case.

Based on this idea as well as in the weakness of the already existing methods of defining the stem form it was decided to use as an alternative in this study the multivariate method of PCA in order to define the stem form and carry out comparisons between control and fertilized trees. Application of this method might lead to a more general expression of the stem form since PCA, as a multivariate method, would provide us with means of considering the general dimensions of variability of the diameters, measured along the stems of the trees, as a whole.

Estimation of stem form of the coniferous trees requires that stems under investigation have evenly spaced measurements of diameter and height from stump to top. Stem curves that have been constructed at tenths of total height can be accurately interpolated to any point of the stem for trees of different DBH and height but having a common taper. Stem curves of this kind can be used not only in the case of the examination of the variation of diameters along the stem, but also for comparisons of the stem curves among different species and size classes. Due to the lack of diameter measurements near the 0.9 of TH position, as well as below 0.1 of TH for all the trees the following examination of stem form applies for part of stem between 0.1 and 0.8 of TH for diameters measured from stem analysis for 1975.

Sets of eight positional diameters at tenths of total height were estimated using linear interpolation of the original diameter measurements taken at each mid-internodal position. Linear interpolation in this case was considered adequate since the original diameter measurements were taken at very close distances. The estimated positional diameters were used to construct one 8x40 data matrix for the control trees and one for the fertilized ones. These two data matrices were fed into the computer, for the estimation of eigenvalues and eigenvectors of the control and fertilized trees respectively. The computer program which was used for the construction of the correlation matrices Tables 30 and 31, the solution of the eigen equations and finally the extraction of eigenvalues and eigenvectors was the subroutine BMDO 1M of the BMD package (ERCC).

In Tables 32 and 33 are recorded the eigenvalues as resulted from PCA of the diameters at tenths of TH. It has already been seen in the previous paragraph that, after the application of PCA and the transformation of the correlated original variates (positional diameters) into uncorrelated ones, the total sum of variance of the original variates is maintained. It has also been seen that the sum of the eigenvalues equals the sum of the variances of the original variates.

An examination of the extracted eigenvalues, of the above Tables, shows that the variance of the first transformed variate has absorbed almost 93% of the total variance. The second variate has absorbed almost 5.5% and the third nearly 1.5% of the total variance (Tables 32 and 33). Thus, the first component accounts for the major part of the variability among the diameters, and the first three components account for more than 99% of the total variance in both cases (control and fertilized trees).

After the extraction of the eigenvalues and eigenvectors the next step in the analysis was the interpretation of the principal components. This step was considered necessary to get to a decision upon the number of new variates having any practical significance to be interpreted. Jeffers (1967) referring to the significance of the principal components suggests that:

"an arbitrary but simpler rule of thumb, which has been proved to be useful in practice, it is to consider only those components which have eigenvalues of 1.000 or greater as having any practical significance"

In the examined cases the first principal component with values 7.401 and 7.419, for the control and fertilized trees respectively,

TABLE 30

CORRELATION COEFFICIENT MATRIX AS RESULTED FROM PCA OF THE CONTROL TREES

	DIAMR1	DIAMR2	DIAMR3	DIAMR4	DIAMR5	DIAMR6	DIAMR7	DIAMR8
DIAMR1	1.0000	0.9917	0.9839	0.9808	0.9736	0.9584	0.8354	0.7273
DIAMR2	0.9917	1.0000	0.9966	0.9932	0.9884	0.9732	0.8585	0.7522
DIAMR3	0.9839	0.9966	1.000	0.9956	0.9931	0.9791	0.8692	0.7585
DIAMR4	0.9808	0.9932	0.9956	1.0000	0.9965	0.9848	0.8906	0.7715
DIAMR5	0.9736	0.9884	0.9931	0.9965	1.0000	0.9918	0.9001	0.7781
DIAMR6	0.9584	0.9732	0.9791	0.9848	0.9918	1.0000	0.9302	0.8117
DIAMR7	0.8354	0.8585	0.8692	0.8906	0.9001	0.9302	1.0000	0.8615
DIAMR8	0.7273	0.7522	0.7585	0.7715	0.7781	0.8117	0.8615	1.0000

TABLE 31

CORRELATION COEFFICIENT MATRIX AS RESULTED FROM PCA OF THE FERTILIZED TREES

	DIAMR1	DIAMR2	DIAMR3	DIAMR4	DIAMR5	DIAMR6	DIAMR7	DIAMR8
DIAMR1	1.0000	0.9895	0.9867	0.9787	0.9700	0.9436	0.8709	0.7074
DIAMR2	0.9895	1.0000	0.9972	0.9897	0.9810	0.9552	0.8904	0.7173
DIAMR3	0.9867	0.9972	1.0000	0.9941	0.9866	0.9646	0.9049	0.7382
DIAMR4	0.9787	0.9897	0.9941	1.0000	0.9958	0.9788	0.9234	0.7542
DIAMR5	0.9700	0.9810	0.9866	0.9958	1.0000	0.9872	0.9360	0.7691
DIAMR6	0.9436	0.9552	0.9646	0.9788	0.9872	1.0000	0.9699	0.8228
DIAMR7	0.8709	0.8904	0.9049	0.9234	0.9360	0.9699	1.0000	0.8909
DIAMR8	0.7074	0.7173	0.7382	0.7542	0.7691	0.8228	0.8909	1.0000

TABLE 32

EIGENVALUES FOR THE AVERAGE TREE FORM

OF THE CONTROL TREES(1975)

COMPONENT NR	EIGENVALUE	PROPORTION OF VARIABILITY %	CUMULATIVE PROPORTION
1	7.40192	92.524	92.524
2	0.43734	5.466	97.990
3	0.11829	1.478	99.468
4	0.02376	0.297	99.765
5	0.01114	0.139	99.904
6	0.00425	0.053	99.957
7	0.00166	0.020	99.977
8	0.00160	0.020	99.997
TOTAL	7.99996		

TABLE 33

EIGENVALUES FOR THE AVERAGE TREE FORM

OF THE FERTILIZED TREES (1975)

COMPONENT NR	EIGENVALUE	PROPORTION OF VARIABILITY %	CUMULATIVE PROPORTION
1	7.41973	92.7469	92.7469
2	0.46444	5.8055	98.5524
3	0.07601	0.9501	99.5025
4	0.01725	0.2156	99.7181
5	0.01383	0.1728	99.8909
6	0.00451	0.0563	99.9472
7	0.00254	0.0317	99.9789
8	0.00166	0.0207	99.9996
TOTAL	7.99997		

absorbed almost 93% of the total variability. The second and third principal components, in both cases had values smaller than 1.000 and absorbed together with the first nearly 99.5 of the total variability of the original variates. Judging from the high percentage of absorbed variability by the first component it is clearly the most significant new variate to be interpreted in this particular case, although the possible interpretation of the next two components bringing the total of the absorbed variability up to nearly 100% might be also considered.

After the examination of the significance of the new variates (principal components) the elements of the eigenvectors corresponding to the first principal components of the control and fertilized trees Table35, were plotted against the positional height (Fig.34). In each case it can be seen that the plottings of the elements of the first eigenvectors result in a characteristic curve which bears similarities with a stem curve. These curves corresponding to the elements of the first eigenvectors might be interpreted as approximations of the mean stem curves of the control and fertilized trees, since the first eigenvector accounted for the linear relationship among the diameters of the trees (Fries and Matern,1966).

The fact that by definition, the new variates (principal components) are mutually independent helps a lot in the interpretation of variability measured by the original variables and thus turns the attention of the researcher on the basic "dimensions" of variability of which his variables are only first approximations (Jeffers, 1967). However the interpretation process is often difficult because the new variates may not have an identifiable separate existance, or may not agree with the existing biological knowledge (Liu and Keister, 1978). It is really one limitation of the PC interpretation that the validity of the

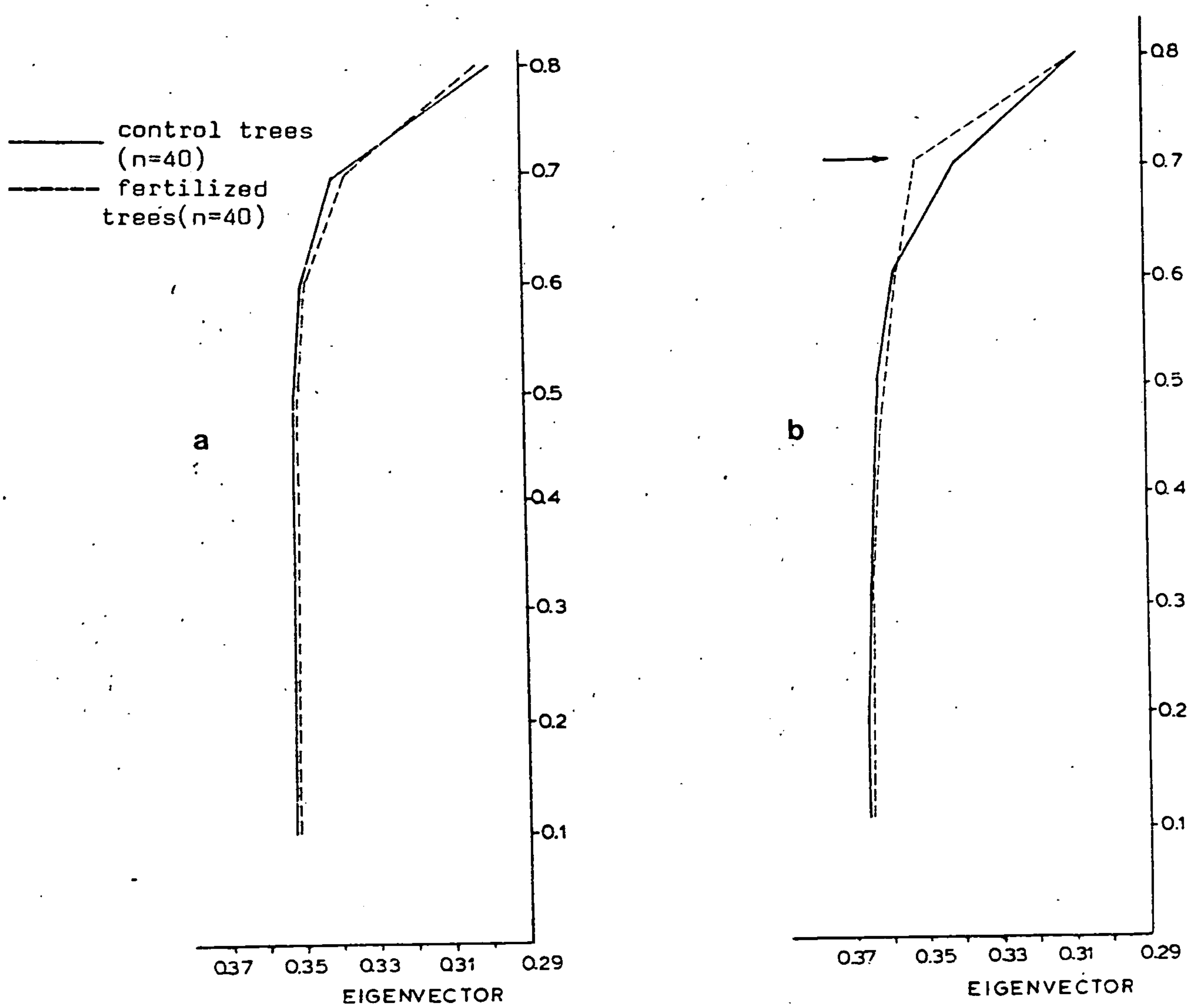


Fig. 34 Plottings of the elements of the eigenvectors against height (a in 1970, b in 1975)

interpretation depends on how the results agree with the general knowledge.

In this case as shown in Table 33 the first eigenvalue which represents the variance of the first new transformed variate (principal component) has absorbed almost 93% of the total variability and therefore pointing that the stem curve of the trees can be characterized almost by the variability of one component only. Also in Figs.(34a) and (b) we have seen that the plottings of the first eigenvector resulted to curves similar to those of stem curves. Therefore the first principal component could be interpreted as an approximation of the mean stem form of the sampled trees and in particular for that part of the stem curve from 0.1-0.8 of TH.

This view is based on:

1. The results of the analysis (high percentage of variability absorbed by the first eigenvector)
2. Agreement with the general biological knowledge (shape of stem curves)
3. Previous reported results (Liu and Keister, 1978)

Fries and Matern (1966) concluded that:

"the first eigenvector gives the linear relationship between the diameters. This is the mean stem form of all the trees. The second eigenvector gives the linear relationship between the deviations from the first etc."

Interpretation of the second and third principal components was not attempted because their eigenvectors (Table34) seemed unstable and changing between control and fertilized trees. Inclusion of more data (diameter measurements) accounting for the lowest part of the stem (butt-swell) might have been of great help in the interpretation of the second or the third component and therefore giving a better insight in the direction of how these two components were attributing

TABLE 34

EIGENVECTORS CORRESPONDING TO THE SECOND AND THIRD PRINCIPAL COMPONENT

	CONTROL TREES		FERTILIZED TREES	
VARIABLES	EIGENVECTORS CORRESPONDING TO COMPONENT NR 2	EIGENVECTORS CORRESPONDING TO COMPONENT NR 3	EIGENVECTORS CORRESPONDING TO COMPONENT NR 2	EIGENVECTORS CORRESPONDING TO COMPONENT NR 3
$D_{0.1}$	0.2840	0.2355	-0.2851	0.4056
$D_{0.2}$	0.2331	0.1759	-0.2641	0.2245
$D_{0.3}$	0.2079	0.1065	-0.2176	0.1657
$D_{0.4}$	0.1655	0.0001	-0.1673	-0.0495
$D_{0.5}$	0.1352	-0.0673	-0.1171	-0.1897
$D_{0.6}$	0.0148	-0.1818	0.0522	-0.3926
$D_{0.7}$	-0.3969	-0.7610	0.3297	-0.5745
$D_{0.8}$	-0.7864	0.5344	0.8053	0.4848

in the formation of stem form.

7.4 COMPARISON OF THE FIRST PRINCIPAL COMPONENT BETWEEN CONTROL AND FERTILIZED TREES

In the previous paragraph it has been seen that the eigenvector corresponding to the first principal component gives the linear relationship between the diameters and it was considered as an approximation of the mean stem form of the sampled trees (Fries, 1965, Fries and Matern, 1966, Liu and Keister, 1978). To trace probable differences in stem form induced by fertilization the elements of the first eigenvector of the control trees were compared with the elements of the corresponding eigenvector of the fertilized trees. These are presented in Table35.

The entries of the above Table showed only minor differences among the elements of eigenvectors between control and fertilized trees at the positions 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.8 of TH. At the position 0.7 of TH there appeared to be a difference 0.0109 between the elements of the elements of the eigenvectors of the control and fertilized trees. This difference compared with the others of the Table was the biggest and therefore indicating that at that part of the stem of the fertilized trees, between 0.6 and 0.8 of TH, their stem curve was improving. Assuming that site and/or genetic factors were equal in both groups of trees this might be attributed to fertilization. If this interpretation is correct it indicates that fertilized trees tend to be more cylindrical. This is in agreement with the evidence presented in Chapter 5, where most of the increment induced by fertilization was found to be concentrated in middle and up to the upper half of the tree stem, and with other reported results (Mitchell and Kellogg, 1972,Wollons and Will, 1975, Whyte and Mead, 1977).

TABLE 35

ELEMENTS OF THE EIGENVECTORS CORRESPONDING TO THE EIGENVALUES OF THE FIRST PRINCIPAL COMPONENT 1975

	CONTROL TREES	FERTILIZED TREES	DIFFERENCE
VARIABLES	COEFFICIENTS FOR COMPONENT V_1	COEFFICIENTS FOR COMPONENT V_1	CONTROL - FERTILIZED
$D_{0.1}$	-0.3650	-0.3643	-0.0007
$D_{0.2}$	-0.3650	-0.3639	-0.0011
$D_{0.3}$	-0.3647	-0.3637	-0.0010
$D_{0.4}$	-0.3630	-0.3620	-0.0010
$D_{0.5}$	-0.3620	-0.3596	-0.0024
$D_{0.6}$	-0.3572	-0.3561	-0.0011
$D_{0.7}$	-0.3407	-0.3516	+0.0109*
$D_{0.8}$	-0.3067	-0.3069	+0.0002

D= the diameters at tenths of total height

*= indicates the biggest difference

At this stage it is worthnoting that it might have been an improvement if more data were available and included in the analysis. Such data could be 3-4 diameter measurements taken from all trees below the 0.1 of TH to account for the butt-swell, as well as diameter measurements to account for the part of the trees above 0.8 of TH. If such data were available it is possible that the first principal component might have absorbed a higher percentage of the variability of the original variates (diameter measurements). That might have accounted also for the linear combinations of the diameters of those parts of the tree stems that were not included in the analysis.

Inclusion in the PCA of such data could have resulted in a re-adjustment through the new scatter of points - which would included the added diameter measurements of butt-swell and the upper part - of the first principal axis (first principal component) in the direction of the biggest variability. The probable consequence of such a re-adjustment might have been a new eigenvector whose elements would give rise to a more realistic curve, closer to the mean stem curve of the examined trees than the ones succeeded from the analysis of the existing data.

So far the analysis was applied to diameters measured and interpolated from stem analysis data for 1975, that is five years following fertilization. A similar analysis was carried out in order to check whether the situation was the same or not at the beginning of the experiment (1970).

In Tables 36 and 37 it can be seen that the cumulative percentage of the absorbed variability of the eigenvectors corresponding to the first three principal components was almost 99.5%. In Table38 as well as in Fig.34a, one can see that the differences between the elements of the eigenvectors corresponding to the first principal component were very small and could be considered negligible at all points

TABLE 36

EIGENVALUES FOR THE AVERAGE TREE FORM

OF THE CONTROL TREES (1970)

COMPONENT NR	EIGENVALUE	PROPORTION OF VARIABILITY %	CUMULATIVE PROPORTION
1	7.63499	95.438	95.438
2	0.21684	2.710	98.148
3	0.10832	1.354	99.502
4	0.02276	0.284	99.786
5	0.01009	0.126	99.912
6	0.00387	0.048	99.960
7	0.00158	0.019	99.979
8	0.00149	0.018	99.997
TOTAL	7.99994		

TABLE 37

EIGENVALUES FOR THE AVERAGE TREE FORM

OF THE FERTILIZED TO BE TREES (1970)

COMPONENT NR	EIGENVALUE	PROPORTION OF VARIABILITY %	CUMULATIVE PROPORTION
1	7.61974	95.247	95.247
2	0.23381	2.922	98.169
3	0.10761	1.345	99.514
4	0.02192	0.274	99.788
5	0.00997	0.124	99.912
6	0.00379	0.047	99.959
7	0.00152	0.019	99.978
8	0.00157	0.019	99.997
TOTAL	7.99993		

TABLE 38

ELEMENTS OF THE EIGENVECTORS CORRESPONDING TO THE EIGENVALUES OF THE FIRST PRINCIPAL COMPONENT (1970)

VARIABLES	CONTROL TREES	FERTILIZED TREES	DIFFERENCE CONTROL - FERTILIZED
$D_{0.1}$	-0.3562	-0.3551	-0.0011
$D_{0.2}$	-0.3554	-0.3540	-0.0014
$D_{0.3}$	-0.3541	-0.3528	-0.0013
$D_{0.4}$	-0.3536	-0.3523	-0.0013
$D_{0.5}$	-0.3527	-0.3516	-0.0011
$D_{0.6}$	-0.3498	-0.3489	-0.0009
$D_{0.7}$	-0.3405	-0.3369	-0.0036
$D_{0.8}$	-0.2976	-0.3005	+0.0029

along the part of the stem curve under examination e.g.0.1-0.8 of TH.

In particular, it can be noted that in 1970, before fertilization could have had an effect, the difference in the eigenvector values at 0.7 of TH was only -0.0036, while in 1975, five years after fertilization, this had changed to +0.0109. This strengthens the view that fertilization has affected stem form.

7.5 A CURVE FITTING METHOD OF THE ELEMENTS OF THE EIGENVECTORS AND MODELS FOR DIAMETER PREDICTION

So far we have examined the application of a multivariate method to define the stem curves of the control and fertilized trees and also the comparison of these stem curves. In this part of the study the intention was to bring the results of PCA into practical application by building up a model for diameter prediction.

From the previous paragraphs we have already seen that:

1. For each eigenvalue there is a corresponding eigenvector and also that these eigenvalues (Tables32and33) decrease rapidly from the first to the fourth, which is already very low, while the decrease from the fourth to the eight is moderate.

2. The cumulative percentage of the first three eigenvalues accounted for almost 100% of the total variability of the original variates (i.e. positional diameters) in both groups of trees.

The above two conditions indicated that the total of variability of the stem curves could be summarized by the first three principal components.

As a consequence of the above it might be expected that if the three new variates (principal components) were included in a modell could estimate quite acurately diameters along the stem of the tree of an average stem form. Fries and Matern (1966) reached the same conclusion when they pointed out that inclusion of the three eigenvectors in one function could result to a system of taper curves for birch.

The sequence steps which were followed for the development of the diameter prediction model can be seen in Fig.35. Steps 1 and 2 dealing with the estimation of the positional diameters as well as with the estimation of the correlation matrix have already been described in the previous paragraph. The data used for the development of the model were the eigenvectors as they resulted from the use of positional diameters measured from stem analyses for 1975.

STEP 3

At this step in order to recognise and bring the results of PCA into practical application a curve fitting method of the elements of the eigenvectors was considered necessary. For this purpose multiple regression analysis was used. The dependent variable was the eigenvector which consisted of eight elements and the independent variables were the positional height and its powers($H^{1/2}$, H, H^2, H^3, H^4, H^5, H^6, H^7, H^8). This procedure was applied for each of the first three eigenvectors. In Tables 39 and 40 are presented the coefficients and statistics of these functions for the control and fertilized trees respectively. In Table 41 are presented the functional values of the eigenvectors as they were estimated by the regression models above, which were of the general form:

$$f_i(V) = \sum_{j=1}^{9} a_{i,j} H^j \qquad i=1,2,3 \qquad (18)$$

where V=the eigenvector

a=the coefficients

H=the positional height

j=the powers of height

Fig. 35 SEQUENCE OF STEPS USED TO DEVELOP A PREDICTION FUNCTION FOR DIAMETER AT TENTHS OF HEIGHT USING P.C.A.

STEP 1

ESTIMATION OF DIAMETERS AT TENTHS OF T.H. (USING LINEAR INTERPOLATION) TO BE USED FOR THE PREPARATION OF DATA MATRIX AS SHOWN ON THE RIGHT

STEP 2

P.C.A.-ESTIMATION OF THE CORRELATION MATRIX AND EXTRACTION OF EIGENVALUES AND EIGENVECTORS.

STEP 3

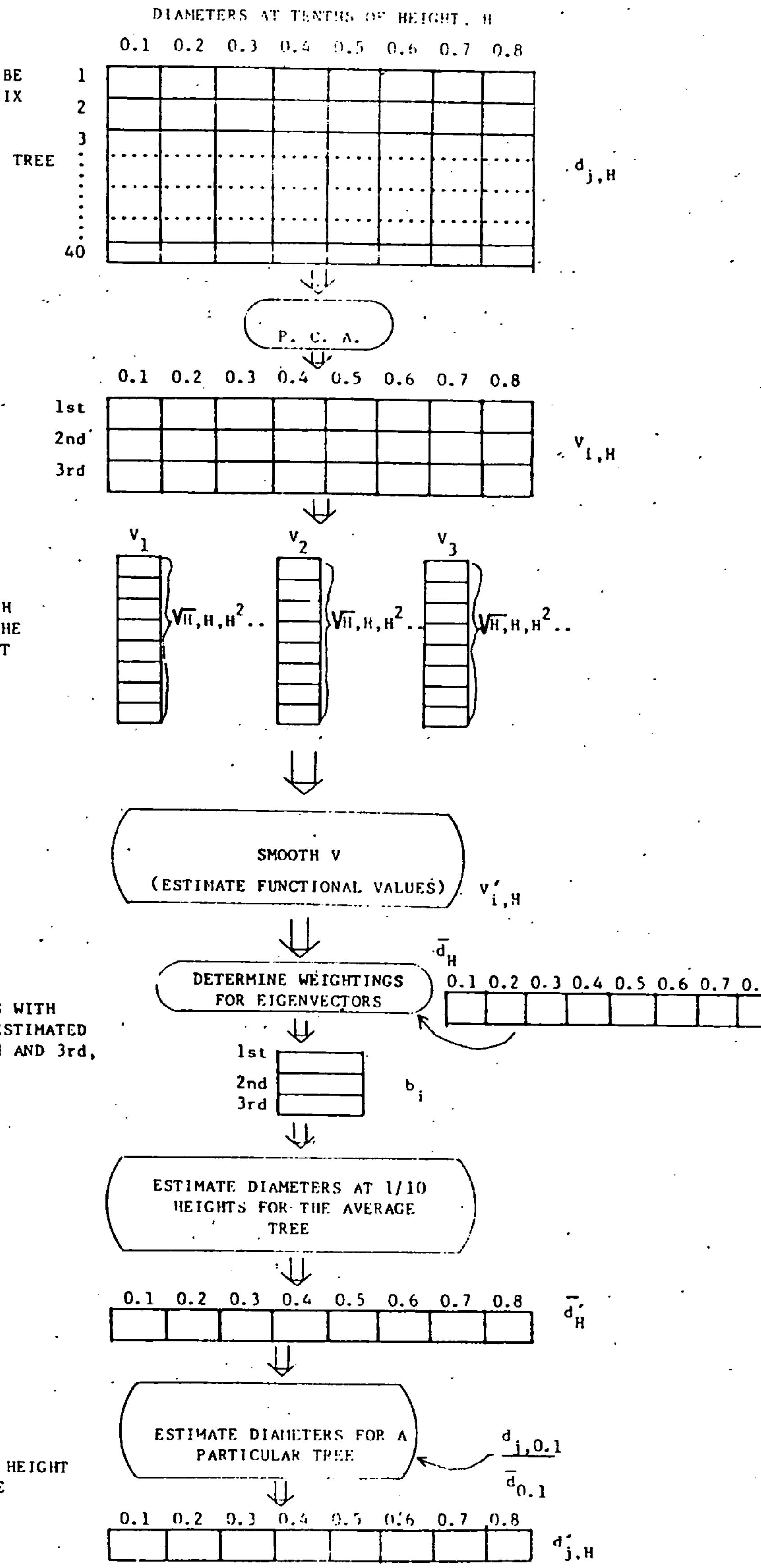
STEPWISE REGRESSION USED AS A CURVE FITTING METHOD OF THE ELEMENTS OF EACH EIGENVECTOR. DEPENDENT VARIABLE WAS THE EIGENVECTOR'S ELEMENTS AND INDEPENDENT THE POSITIONAL HEIGHTS AND THEIR POWERS: $\sqrt{H}, H, H^2, H^3, H^4, H^5, H^6, H^7, H^8$

STEP 4

MULTIPLE REGRESSION OF MEAN DIAMETERS WITH THE FUNCTIONAL VALUES (AS THEY WERE ESTIMATED IN THE PREVIOUS STEP) OF THE 1st, 2nd AND 3rd, EIGENVECTORS

STEP 5

ESTIMATION OF DIAMETERS AT TENTHS OF HEIGHT FOR A PARTICULAR TREE BY APPLYING THE ADJUSTMENT FACTOR.



SYMBOLS

H	-	1/10 of total height (0.1,0.2,....0.8)
j	-	tree subscript (j=1,40)
i	-	eigenvector (i=1,3)
$d_{j,H}$	-	positional (interpolated) diameter at 1/10 height for individual trees
$d_{j,0.1}$	-	estimated diameter at the first 1/10 height for individual trees
d_H	-	positional (interpolated) mean diam. at 1/10 heights
$\bar{d}'_{0.1}$	-	estimated mean diam. at the first 1/10 heights
$V_{i,H}$	-	eigenvector elements
$V'_{i,H}$	-	smoothed eigenvector elements
b_i	-	weightings for the first three eigenvectors

TABLE 39

FUNCTIONS OF EIGENVECTORS

COEFFICIENTS AND STATISTICS FROM REGRESSION MODELS

USED TO FIT THE EIGENVECTORS

A. CONTROL TREES

EIGENVECTOR NUMBER	CONSTANT	H	H	H^2	H^3	H^4	H^5	H^6	H^7	H^8	R^2
1	-0.3549	-	-0.01069	-	-	-0.2714	-	-	2.794	-2.4855	0.99
2	0.05708	-	0.9215	-7.266	-	58.87	-81.67	-	-	33.56	0.99
3	0.7267	-	-1.427	-	-	17.89	-	-	-231.53	252.13	0.96

TABLE 40

FUNCTIONS OF EIGENVECTORS

COEFFICIENTS AND STATISTICS FROM REGRESSION MODELS

USED TO FIT THE EIGENVECTORS

B. FERTILIZED TREES

EIGENVECTOR NUMBER	CONSTANT	H	H	H^2	H^3	H^4	H^5	H^6	H^7	H^8	R^2
1	-0.3508	-0.06035	-	0.07905	-	-	-	-0.4703	-	1.0065	0.99
2	-0.4018	-	0.3709	-0.4991	-	2.538	-	-	-	0.9304	0.99
3	0.8388	-	1.358	-	-	3.497	-	-	-87.54	106.00	0.99

TABLE 41

FUNCTIONAL VALUES OF EIGENVECTORS ESTIMATED BY REGRESSION MODELS

	CONTROL TREES			FERTILIZED TREES		
HEIGHT	1st EIGENVECTOR	2nd EIGENVECTOR	3rd EIGENVECTOR	1st EIGENVECTOR	2nd EIGENVECTOR	3rd EIGENVECTOR
0.1	-0.3673	0.2809	0.2772	-0.3646	-0.2892	0.4097
0.2	-0.3619	0.2466	0.1148	-0.3635	-0.2518	0.2362
0.3	-0.3656	0.1885	0.0559	-0.3636	-0.2229	0.1111
0.4	-0.3625	0.1700	0.0620	-0.3620	-0.1815	-0.0045
0.5	-0.3600	0.1504	0.0118	-0.3597	-0.1020	-0.1727
0.6	-0.3583	-0.023	-0.3066	-0.3560	0.0503	-0.4300
0.7	-0.3422	-0.3889	-0.7045	-0.3516	0.3269	-0.5564
0.8	-0.3067	-0.7869	0.5231	-0.3029	0.8061	0.4819

STEP 4

Since the maximum of total variability was absorbed by the first three significant variates it was considered that for a simultaneous evaluation of the three components a linear combination of the previously described functions of eigenvectors would provide us with the desired results for diameter prediction. In other words the model needed for this purpose required a way to evaluate the weighting factors for each of the three significant components. This model would be able to estimate diameters along the stem of the trees with average stem form.

Again multiple regression was used for this purpose. The data used as dependent variables were the interpolated diameters and as independent variables were used the functional values of the previously derived functions of eigenvectors (Table 42). The general model was of the form:

$$f(d_j) = \sum_{i=1}^{3} a_i f(v_i) \qquad (19)$$

Coefficients and statistics for the models of the control and fertilized trees are given in Table 42 . The above model could be used for the diameter prediction of the tree with mean stem form.

In the next Table(43)are presented the actual (interpolated) diameters (at tenth of TH) as well as the predicted ones from equation for the average of the control and the fertilized group respectively. The differences in both cases are small and actually less than 0.5 cm. These differences conform to the average trees.

STEP 5

Finally an adjustment factor was needed for the estimation of the diameters of trees with different than the average form. Such a factor, among others (e.g. DBH) was considered to be the ratio of

TABLE 42

COEFFICIENTS AND STATISTICS FROM MULTIPLE REGRESSION MODELS USED FOR DIAMETER PREDICTION

A. CONTROL TREES

CONSTANT	V_1	V_2	V_3	R^2	SE	RMS
103.45	261.94	21.66	2.21	0.99	0.359	0.129

B. FERTILIZED TREES

CONSTANT	V_1	V_2	V_3	R^2	SE	RMS
221.62	593.02	-37.73	-12.98	0.99	0.235	0.055

V_1, V_2, V_3 = the first, second and third eigenvectors, correspondingly

RMS = Residual Mean Square

SE = Standard Error

TABLE 43

ACTUAL AND PREDICTED DIAMETERS AND THEIR DIFFERENCE

ACTUAL MEAN DIAMETERS (cm)

HEIGHT	CONTROL DIAMETER	FERTILIZED DIAMETER
0.1	15.12	16.14
0.2	13.91	14.94
0.3	12.78	13.80
0.4	11.46	12.65
0.5	10.35	11.50
0.6	8.89	9.93
0.7	7.11	7.91
0.8	4.87	5.33

PREDICTED DIAMETERS FROM PCA (cm)

HEIGHT	CONTROL DIAMETER	FERTILIZED DIAMETER
0.1	15.07	16.09
0.2	14.23	14.74
0.3	12.46	13.91
0.4	11.22	12.96
0.5	10.47	11.50
0.6	9.27	9.68
0.7	7.06	8.00
0.8	4.91	5.32

DIFFERENCE (ACTUAL - PREDICTED) (cm)

HEIGHT	CONTROL DIAMETER	FERTILIZED DIAMETER
0.1	0.05	0.05
0.2	-0.32	0.20
0.3	0.32	-0.11
0.4	0.24	-0.31
0.5	-0.12	0.00
0.6	-0.38	0.25
0.7	0.05	-0.09
0.8	-0.04	0.01

diameter at 0.1 of TH of the tree to the average diameter of all the trees in the sample at the same position (0.1 of TH). This factor was prefered because as compared with another factor that might be used, e.g. the DBH of the tree, had a better "biological" meaning since it was related to the total height of the tree and DBH on the contrary is usually a rather "technical" measure of convenience.

After the introduction of the aformentioned adjustment factor the final model to be used for diameter prediction was of the form:

$$D(x) = Rf(d) \qquad (20)$$

where R is the adjustment factor and

f(d) is that from equation (19)

The use of above model for the prediction of diameters, a total of 8x40=320 diameter measurements for the control and 8x40=320 for the fertilized trees, gave differences that ranged as in the Table below:

TABLE 44

RANGE OF DIFFERENCES BETWEEN ACTUAL AND PREDICTED DIAMETERS

	BELOW 1 cm	BETWEEN 1-2 cm	ABOVE 2 cm
CONTROL TREES	290	29	1
FERTILIZED TREES	293	26	1
TOTAL	583	55	2

Finally it was decided to examine how well the models fit the data. For this purpose two correlation analyses were used, one for the control and one for the fertilized trees. The interpolated (positional) diameters for a random sample of 20 control trees (a total of 160 diameters) were correlated with their predicted values as they were estimated using equation (20). The correlation coefficient was 0.991. For a similar sample of fertilized trees (20 trees, 160 diameters) the correlation coefficient was 0.982. In both cases the correlation coefficients were very high indicating that both functions developed here could be applied with reliability as models for diameter predictions.

7.6 DISCUSSION

As it has been seen at the beginning of this Chapter definition of stem form is a difficult task because of the great number of factors contributing to the variability of its development. The big number of papers dealing with this subject (Larson, 1963, Sterba, 1980) proves the difficulty of coping with it. The multivariate method (PCA) was used in this study as a means of looking rather for general "dimensions" of variability for the solution of this problem, -stem form identification'- than adopting the variables influencing the stem form one at a time in a model, or even force the trees to grow conforming to the dimensions of paraboloids, neiloids or conoids. The data reduction that was succeeded by applying this method - that is only three principal components accounted for the total variability of the diameters - coupled with the good results of the predicted diameters, when these three principal components were combined into a model, proves the applicability of the method.

The main limitation of this method should be noted here also. This is the difficulty that the researcher is faced with when he attempts to interpret the principal components which in many cases are very difficult to be identified separately. In such cases experience gained from previous results combined with good knowledge of the variables helps in the interpretation of the results provided that the principal components describe the original variance in as small a number of uncorrelated dimensions as possible.

Application of PCA in this study did not imply that this was the best method to be applied to the purposes set in this Chapter. It was used here because of its main principle as a multivariate method which integrates the various sources introducing variability in the different biological problems - like the one of the stem curve - which ideally should not have been separated and also because of its function of "generating hypotheses" (Pearce, 1969). Indeed comparison of the elements of the eigenvectors of the control and fertilized trees indicated about the stem curve at the upper half of the stem of the fertilized trees was improving and hence "pointing" the direction for further research. In this sense PCA can be considered as a method seeking for more general expression of the variables under examination without oversimplifying things and hence not bearing the consequences of such a simplification that might influence the results.

As compared with other methods dealing with stem form also, PCA applied for diameter prediction might seemed complicated. But equally complicated might be considered most of the recent methods dealing with this subject and incorporating for its quantification "segmented polynomial analysis" when using two different polynomials "to be joined" at the inflection point of the stem curve (Demaerschalk and Kozak, 1977) and/or using non linear least squares to define the lengths of the parts of the stem for which different polynomials would be applied (Max and Burkhart, 1976).

The use of computers in recent years has made easy the application of complicated methods of mathematical analysis as is indicated by the frequency of their use. This fact of course should not be considered as a proof that complicated methods are the best to be used since simple methods might also be used to adequately define the stem form.

To reach a conclusion it is the purpose that has been set and the effort required balanced to precision which one needed to define the particular method to be used in order to enable a decision to be made upon the functional values of the stem form.

7.7 SUMMARY AND CONCLUSIONS

A multivariate method (PCA) used in this Chapter had two purposes:

1. To define the stem form of Sitka spruce treesso that comparisons of stem form comparisons between control and fertilized trees could be carried out, and

2. To develop diameter prediction functions for the above trees.

The data used for the above purposes were derived from diameter measurements taken at each mid-internodal position using linear interpolation.

The main characteristic of PCA is to transfer all the variance and covariance existing among the original variates (in this case th positional diameters) to a smaller number of new variates which are uncorrelated and absorb almost all the above variance. The new variates (principal components) are independent and therefore can be examined separately. In our case the first three principal components absorbed almost 100% of the variability existing among the diameters and therefore could be considered as summarizing the total variability of the stem form of the examined trees. The first eigenvector absorbed nearly 93% of the total variability and was considered the most significant component to be interpreted. Based on the plotting of the eigenvectors as well as on previous reported results, it was interpreted as a similitude of the stem curve of the trees. Comparison of the elements of the eigenvectors corresponding to the control and fertilized trees indicated that the latter were improving their stem curve in the part between 0.6-0.8 of total height. Inclusion of more data helping the interpretation of the second and third eigenvectors would have contributed to a better way of stem form definition and therefore probably to more indications of differences between treated and untreated trees.

Finally combination of the three significant components in one model gave very good results in the prediction of diameters along the stem of the trees. This can be considered as an indirect proof that the stem form of the trees could be adequately described by using only the three significant components as well as the successful application of the multivariate method for the stem form definition.

CHAPTER 8

STEM FORM COMPARISON USING RELATIVE DIAMETERS

8.1 INTRODUCTION

In the previous Chapter as well as in Chapter 6 we have seen that there was evidence that the stem curve of the fertilized trees was different than the one corresponding to the control trees. In this Chapter in a further attempt to quantify these probable differences relative diameters were used (Hohenadl 1922, Assmann, 1970). The reason for using relative diameters was based upon an early investigation done by Prodan (1944) who used Hohenadl's method and concluded that relative stem curves corresponded better with the mechanical and physical premises than do polynomials in general (Sterba, 1980).

According to Assmann, Hohenadl in his method for volume estimation used diameters taken at the same relative positions of the stem (tenths of TH). As a reference diameter he used the diameter at the first tenth of TH measured from ground level.

However it is likely that the use as a reference diameter of the one taken at 0.2 of TH gives better results. First Assmann considers that:

"even better information about changes in stem form could be obtained if a sectional area above the point of inflection of stem curve (and thus above the base of the stem) approximately at 0.2 or 0.3 measured from base could be chosen as a reference point because the zone of the steadiest increment lies at approximately one third of stem height measured from base".

Second J. Christie (personal communication) concluded that the variation in stem increment at any point is least at between 0.1 and 0.3 of TH. Finally Kozak and Demaerchalk (1977) studying differences in stem shape of trees belonging to different species as well as within the same species, found that the inflection point (the point where the stem curve changes from neiloid to paraboloid) seems to be at a more or less constant relative height. This relative height ranged from 20-25% of the TH measured from the ground level.

From all the above it was concluded that the reference diameter to be used in this study for the estimation of the relative diameters, was the diameter at 0.2 of TH measured from the base of the stem.

8.2 STEM FORM COMPARISON BETWEEN CONTROL AND FERTILISED TREES

Having estimated the diameters of trees at tenths of TH (Chapter 7) the relative diameters were estimated using as a reference diameter the one at 0.2 of TH (relative diameter:the ratio of each diameter to the diameter at 0.2, expressed as a percentage). In Figs 36 and 37 are presented the relative mean stem curves of the 40 control and 40 fertilized trees in the years 1970 and 1975 respectively. A comparison of the stem curves in these two Figures is informative in several aspects. In Fig.36 can be seen that the resulting stem curves (plots of relative diameters) were found to possess a quite similar pattern in both samples taken in the beginning of the experiment. It can also be seen that there are only minor differences between the relative stem curves of the control and fertilized trees. In Fig.37 it is clear that following fertilization the situation has changed. This time the relative stem curve of the fertilized trees has over-taken the stem curve of the control ones and there were clear differences between them above a certain height.

The differences at the positions 0.1, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8 for the relative stem curves in 1970 were tested using the t-test. Table45 shows the average relative diameters and the results of the test.

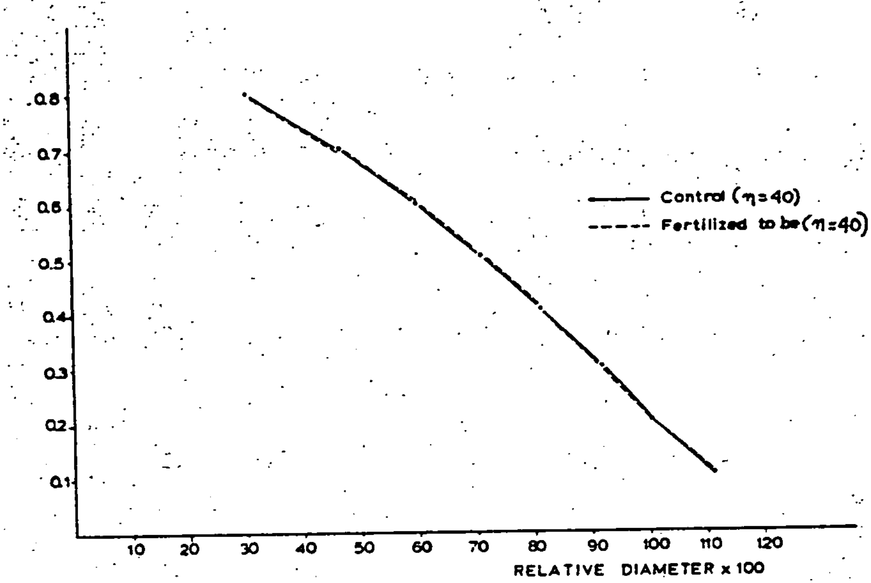


Fig. 36 Average reeative stem curves in 1970

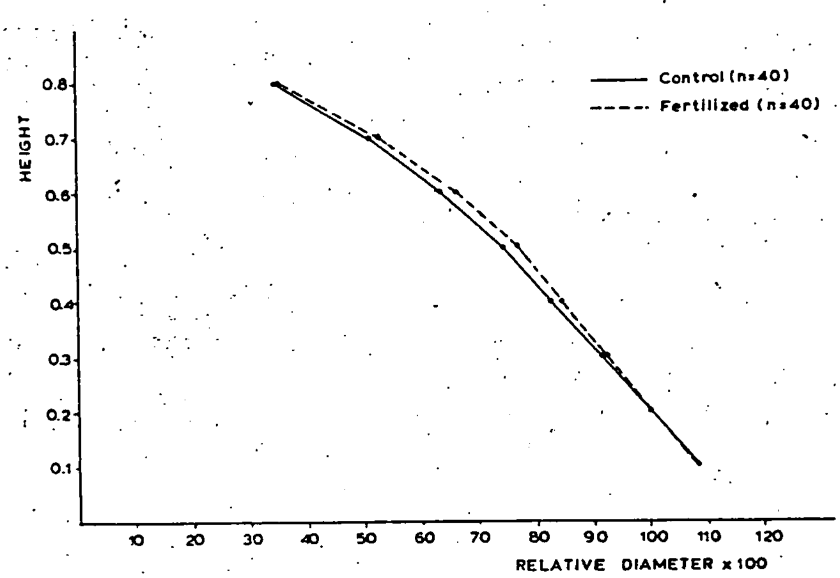


Fig. 37 Average relative stem curves in 1975

TABLE 45

AVERAGE RELATIVE DIAMETERS IN 1970,

THEIR DIFFERENCE AND TEST FOR SIGNIFICANCE

HEIGHT (in tenths)	RELATIVE DIAMETERS x 100		CONTROL-FERTILIZED DIFFERENCE	TEST FOR SIGNIFICANCE (t-test)
	CONTROL	FERTILIZED		
0.1	110.90	111.40	-0.50	n.s
0.2	100.00	100.00	0.00	-
0.3	92.00	91.60	+0.40	n.s
0.4	81.50	81.60	-0.10	n.s
0.5	70.80	70.90	-0.10	n.s
0.6	52.60	59.90	-0.30	n.s
0.7	46.40	46.90	-0.50	n.s
0.8	31.00	30.40	+0.60	n.s

NOTES

1. As a reference diameter for the estimation of the relative diameters the diameter at 0.2 of TH was used.

2. n.s. No significant difference at $p < 0.05$

3. No of trees tested: 40 control and 40 fertilized

From the above results it can be concluded that in 1970 there were no differences in the relative stem curves between control and trees to be fertilized.

The corresponding situation in 1975 is shown in Table 46.

TABLE 46

AVERAGE RELATIVE DIAMETERS IN 1975,

THEIR DIFFERENCE AND TEST FOR SIGNIFICANCE

HEIGHT (in tenths)	RELATIVE DIAMETERS x 100		CONTROL-FERTILIZED DIFFERENCE	TEST FOR SIGNIFICANCE (t-test)
	CONTROL	FERTILIZED		
0.1	108.60	108.10	+0.50	n.s
0.2	100.00	100.00	-	-
0.3	91.80	92.40	-0.60	n.s
0.4	82.30	84.70	-2.40	s.s.
0.5	74.30	77.00	-2.70	s.s.
0.6	63.90	66.50	-2.60	s.s.
0.7	51.10	53.00	-1.90	n.s.
0.8	35.00	35.70	-0.70	n.s.

NOTES

1. n.s.-No significant difference at $p < 0.05$
2. s.s.-statistically significant at $p < 0.05$
3. No of trees tested: 40 control and 40 fertilized

The results of the above table indicate that there were statistically significant differences in the average relative stem curves of the control and fertilized trees at the positions 0.4, 0.5 and 0.6 of the TH caused probably by fertilization. Finally a further examination was carried out to examine the responses of the relative stem curves of trees in different dominance classes.

The trees were grouped in three groups according to their position in the stand canopy (Chapter2) as follows:

Group 1 - suppressed and subdominant trees

Group 2 - co-dominant trees

Group 3 - dominant trees

(a total of six groups, three control and three fertilized).

The relative average stem curves of the three groups, both control and fertilized, are presented in Figs 38,39,40 /.The same pattern in the stem curves appear to exist. There also appear differences in the relative stem curves berween control and fertilized trees which are greater for the trees belonging to the groups 2 and 3 indicating that fertilization was affecting the stems of the trees belonging to the co-dominant and dominant classes.

The differences at the positions 0.1-0.8 were tested using the t-test,(Tables 47,48,49).

The results of these **Tables** can be summarized as follows:

1. There were no statistically significant differences between the average relative stem curves of the control and fertilized trees of group 1.

2. There were statistically significant differences at the postions 0.4, 0.5 and 0.6 of TH between the average relative stem curves of the control and fertilized trees of group 2.

3. There were statistically significant differences at the positions 0.3, 0.4, 0.5, 0.6 and 0.7 of TH between the average relative stem curves of the control and fertilized trees of group 3.

8.3 SUMMARY

From all the above analysis it can be concluded that fertilization resulted in an increased increment over the middle part of the stem in average terms. Grouping of the trees into the three dominance classes revealed that co-dominant and dominant trees were the most benefited ones. In the case of co-dominant trees, both diagrams and t-test showed,

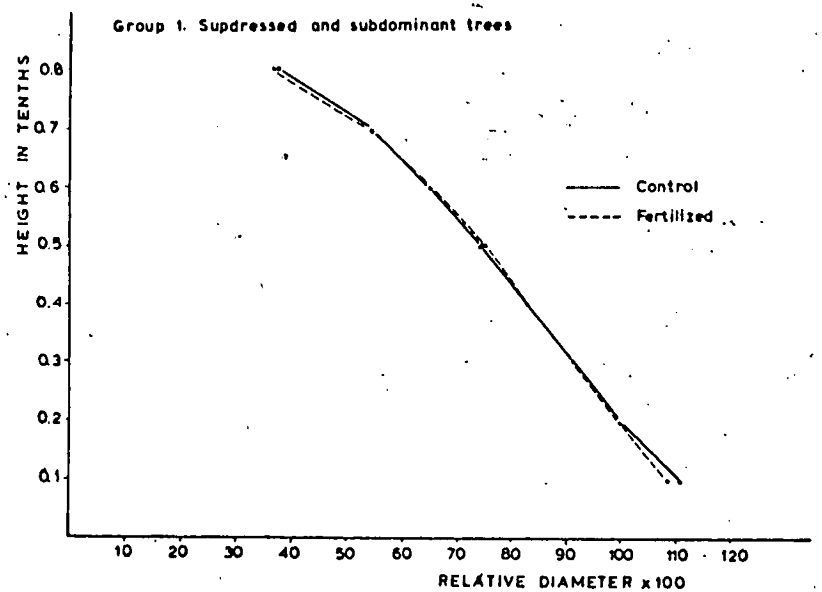


Fig. 38 Average relative stem curves (1975)

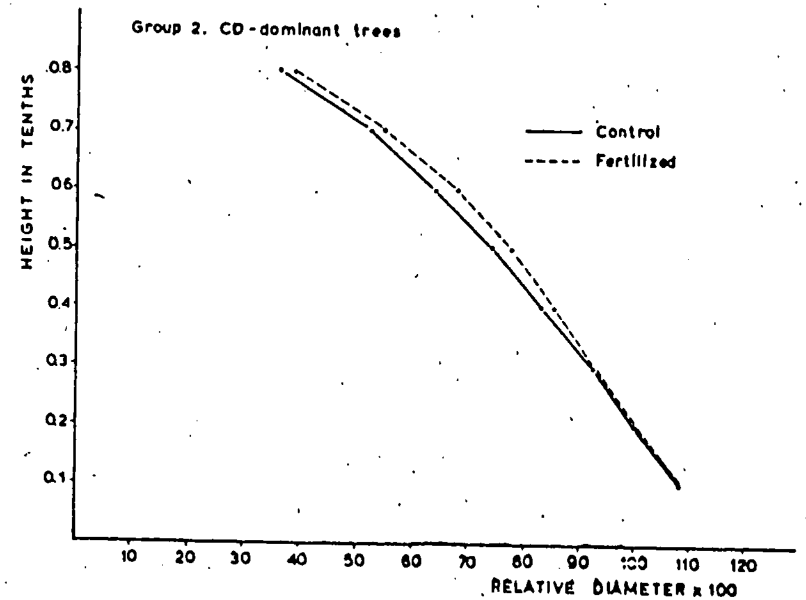


Fig. 39 Average relative stem curves (1975)

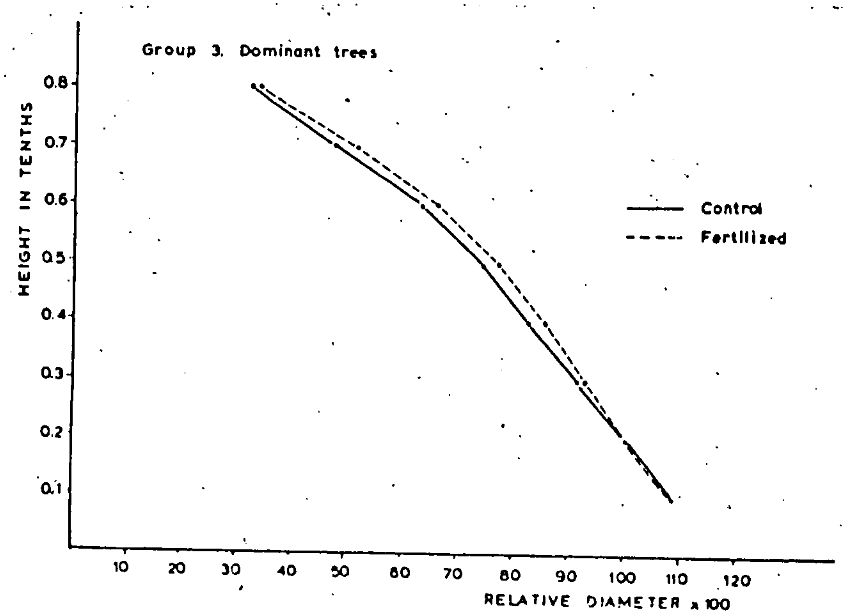


Fig. 40 Average relative stem curves (1975)

TABLE 47

AVERAGE RELATIVE DIAMETERS (1975)

GROUP 1 - SUPPRESSED AND SUBDOMINANT TREES

HEIGHT (in tenths)	RELATIVE DIAMETERS x 100		DIFFERENCE CONTROL-FERTILIZED	TEST FOR SIGNIFICANCE (t-test)
	CONTROL	FERTILIZED		
0.1	110.55	108.94	1.61	n.s.
0.2	100.00	100.00	-	-
0.3	91.57	91.18	0.39	n.s.
0.4	82.87	82.80	0.07	n.s.
0.5	74.62	75.07	-0.45	n.s.
0.6	64.81	65.27	-0.46	n.s.
0.7	54.31	53.56	0.75	n.s.
0.8	37.50	36.20	1.30	n.s.

NOTES

1. n.s.-no significant difference at $p \leqslant 0.05$
2. No of trees tested: 12 control and 12 fertilized

TABLE 48

AVERAGE RELATIVE DIAMETERS (1975)

GROUP 2 - CO-DOMINANT TREES

HEIGHT (in tenths)	RELATIVE DIAMETERS x 100		DIFFERENCE CONTROL-FERTILIZED	TEST FOR SIGNIFICANCE (t-test)
	CONTROL	FERTILIZED		
0.1	108.32	108.59	-0.27	n.s.
0.2	-	-	-	-
0.3	92.20	92.62	-0.42	n.s.
0.4	83.11	85.10	-1.99	s.s.
0.5	74.28	77.96	-3.68	s.s.
0.6	64.20	67.90	-3.70	s.s.
0.7	52.72	54.85	-2.13	n.s.
0.8	36.13	38.94	-2.81	n.s.

NOTES

1. n.s. -no significant difference at $p \leqslant 0.05$
2. s.s. -statistically significant at $p < 0.05$
3. No of trees tested: 14 control and 11 fertilized

TABLE 49

AVERAGE RELATIVE DIAMETERS (1975)

GROUP 3 - DOMINANT TREES

HEIGHT (in tenths)	RELATIVE DIAMETERS x 100		DIFFERENCE CONTROL-FERTILIZED	TEST FOR SIGNIFICANCE (t-test)
	CONTROL	FERTILIZED		
0.1	107.96	107.50	0.46	n.s.
0.2	100.00	100.00	-	-
0.3	91.59	92.79	-1.20	s.s.
0.4	82.72	85.30	-2.58	s.s.
0.5	74.15	76.99	-2.84	s.s.
0.6	63.26	66.20	-2.94	s.s.
0.7	47.56	51.53	-3.97	s.s.
0.8	32.30	33.60	-1.30	n.s.

NOTES

1. n.s.-no significant at $p < 0.05$
2. s.s.-statistically significant at $p < 0.05$
3. No of trees tested: 14 control and 17 fertilized

that the effect was biggest in the middle part of the stem. In the case of the dominant trees the effect was found to be more extensive than in the case of the co-dominants and therefore covering a larger part of the stem (the differences from 0.3-0.7 were statistically significant).

Finally in the case of suppressed and subdominant trees the effect of fertilization was not found to be significant due probably to the disadvantage of being subjected to suppression by the crowns of co-dominant and dominant trees in the stand.

The above results are in concordance with the results of the examination of the internal pattern of increment as it was redistributed after fertilization (Chapter 5).

CHAPTER 9 SUMMARY AND CONCLUSIONS

9.1 DISCUSSION OF METHODS FOR VOLUME ESTIMATION OF THE TREES

In this study the response of pole stage Sitka spruce trees to PK fertilization was examined. The volume response of the trees is the most important response but, because fertilization affects the shape of the tree, volume tables do not necessarily give good results in cases of fertilization (Chapter 1).

In the preceding chapters a number of methods relevant to volume estimation were presented. Some of these methods dealt with the relationship between diameter and height but are basically related to volume estimation by considering the resulting solid of revolution.

The methods can be arranged as follows:

1. Whole-stem functions

These could be divided into theoretical and empirical. Theoretical functions are based on different theories such as Metzger's theory or Gray's theory (Chapter 4). Both theories are mainly based on the assumption that the wind action was the main factor influencing the stem form development, and since trees were acting as "beams of uniform resistance to bending", the stem was bound to approach the dimensions of either a cubic paraboloid (Metzger) or a quadratic paraboloid (Gray). The above theories results in a relationship between height and diameter. Having established a theoretical relationship between diameter and height, the volume of the tree can be estimated by integration. These methods have the advantage that few measurements on each tree are needed.

However, the tree stem is a component of a living organism and because of that it is exposed to influences of a great number of external factors that may influence tree growth. Therefore stem form is likely to be a more complex function of height than these theories would suppose, and the diameter:height relationship suggested by the described theories is probably inadequate.

A great number of empirical mathematical functions have been developed for the description of stem curves (Chapter 7) using polynomials, combined logarithmic and polynomial functions, etc. Having established a function that describes the stem curve, it is possible to integrate for the total height of the tree to estimate its volume. Such functions have again the advantage of using a small number of diameter measurements on each tree for its volume estimation. However, the great number of such functions that has been developed so far proves the difficulty of this task. This might be attributed in a way to the fact that the recorded stem diameters present irregularities due to errors of observations and also to actual irregularities, and as a consequence the recorded diameters never agree exactly with mathematical functions.

For the mathematical expression of stem form multivariate methods have also been used. Such a method - Principal Component Analysis - was first used by Fries and Matern (1966). Liu and Keister (1978) used P.C.A to compare stem curves of different species and also to estimate the volume of the trees.

Principal Component Analysis was also used in this study (Chapter 7) to express the stem form of the control and fertilized trees, so that comparisons could be carried out, to detect the influence of fertilization on the stem form of the trees.

2. <u>Sectional methods</u>

The early observed similarity of tree stem with solids of revolution (Spurr 1952; Husch, 1963) gave rise to the use of formulae developed originally for stereometric solids to be applied for the volume estimation of the tree stem.

According to the sectional method of volume estimation (Chapter 4), the stem of a tree is divided into sections, the volume of each section is estimated and by summing up the volumes of the sections the stem volume is finally obtained.

Based on the above principle - that is stem sections approach the dimensions of stereometric solids - several formulae were applied for the volume estimation of the sections (Huber's formula, Smalian's formula, Newton's formula). Usually for the application of a sectional method for volume estimation, the stem is divided into sections of equal length (tenths of total height, three or two metre sections), and one or two diameters with the length of the section are used for the volume estimation of the section (Huber's or Smalian's formula). As long as the stem sections approach the dimensions of paraboloids, these formulae will give good results. But in such cases, it is possible that the positions where the diameter measurements will be taken will coincide with whorl nodes, stem irregularities, etc, and thus either causing errors in the recorded diameters or necessitating double measurements (e.g. above and below the position of irregularity and taking the average). On the other hand, tree stems are capable of assuming an infinite variety of shapes (Grosenbaugh, 1964). Finally, it is known that there are numerous points of inflection on the stem curve (Assmann, 1970; Grosenbaugh, 1964). Because of all these and since fertilization might add to this situation by changing the stem curve at unknown points, a sectional method for the volume estimation of the trees was developed (Chapter 4), that might cope with changes in stem shape.

The proposed method for the sectional volume estimation which was developed in this study would be able to incorporate any changes in stem shape since the diameter measurements are taken close enough. Therefore any changes in the shape of sections would only cause very small discrepancies (less than 1%) among the main three formulae (for conoid, paraboloid or neiloid frustrum) used for the volume of the section, provided that the difference between the two ends of a section is less than 2.5cm. For each particular situation the diameter step which will be judged as adequate for the degree of discrepancy (accuracy) and number of measurements (effort) can be selected.

Application of this method is not considered to be the best, since the diameter step will need readjustments in order to retain the same accuracy, and this might involve extra work. As compared with the other methods this method has the disadvantage that it needs even more diameter measurements to be taken for the volume estimation. For experimental purposes, when accuracy is needed, such as cases of volume response estimation involving probable changes in stem shape, this method in its strict form (diameter step of 2.5cm) could be used and this would increase the number of diameter measurements (effort). Still more work is needed in order to compare the results of this method - in terms of effort and accuracy - with the usual sectional methods used (e.g. stem diameter measurements every three meters, or in tenths of total height and use of Smalian's or Huber's formula). Obtaining such comparative results will either justify the use of this method for general purpose use, or will restrict its use only for pure scientific purposes because of the increased effort needed for its application.

9.2 DISCUSSION OF THE RESULTS

The results presented in this study indicate that following PK fertilization the stem form of trees changed. Fertilized trees underwent changes as the result of the redistributed growth over the stem of the trees.

At the beginning of the experiment two treatments were carried out in the experimental area:

1. Fertilization, and

2. Thinning

Therefore any difference between the control (thinned) and the fertilized (thinned) trees might be attributed to fertilization.

Examination of the internal pattern of growth (Chapter 5) revealed that following fertilization ring width and/or ring area increased dramatically over the stem of the trees. These results were indicated in the longitudinal direction (ring sequences no. 1),in the radial direction (ring sequences no. 2), and in the radiolongitudinal direction (ring sequences no. 3). Finally in the contour diagrams the situation was presented even better.

Examination of the ring sequences no. 3, which reflect the prevailing site conditions in the stand (Duff and Nolan, 1957; Richardson, 1961) indicated that from 1956 onwards there was a slight decline in the site conditions of the experimental area. This result might be attributed to the lack of nutrients. This was confirmed indirectly since following fertilization a characteristic upwards trend appeared in the no. 3 sequences, denoting the improvement of the site conditions.

Statistical analysis of the results in terms of basal area, cross-sectional area at half of total height, total height and form factor, during the five year period examined, showed statistically significant differences between control and fertilized trees. Basal area was increased by 56% over the control increment during the period under examination, and cross-sectional area, at half total height, was increased by 74% over corresponding controls. This was the first indication that fertilized trees were improving in cylindricity by showing a greater rate of growth at half of total height than in the breast height position (74% as compared with 56%). During the same period height increment was estimated as 53% over the corresponding control increment.

Examination of the form factor change - defined as the difference in form factor at the beginning and at the end of the period examined - indicated that there were statistically significant differences between control and fertilized trees. This confirmed the higher per cent increase of the cross-sectional area increment at half total height. Despite the fact that both basal area and height were responding to treatment at a more or less equal per cent rate the stem shape was changing.

This result might probably be attributed to an improvement in the photosynthetic capacity and/or photosynthetic efficiency of the trees (Brix, 1976) which caused the higher rate of growth. Measurements of the leaf area index of the trees of this experiment (Malcolm, pers. com.) showed that there were statistically significant differences between control and fertilized trees (leaf area index was 5.7 for the control and 8.5 for the fertilized trees).

Examination of the stem curves using Principal Component Analysis showed that fertilized trees were improving their stem form - in average terms - in the upper half of the stem. Application of this method also indicated that perhaps better results could be achieved, if more diameter measurements were included in the model, accounting for the lower and also for the upper part of the stem. The good results of diameter prediction, when the three principal components were included in one model proved indirectly the applicability of this method for such purposes. The weakness of the first principal component for a complete description of the stem curve (it absorbed 93% of the total variability of the diameters in this study), might be considered due to the fact that the second and third components participate also in the stem curve description. These two components might account perhaps for the discrepancies induced by the crown length and butt-swell on the stem curve (Fries and Matern, 1966).

Average relative stem curves showed that the dramatically increased growth rate of the fertilized trees, observed in the tree diagrams of Chapter 5, was affecting the relative stem curves in the middle part of the stem (statistically significant differences at the 0.4, 0.5 and 0.6 tenths of total height).

It is worth noting that principal component analysis indicated that the change in the average stem curves occurred well in the upper half of the stem while the average relative stem curves showed that the change occurred in the middle part of the stem. Three findings are relevant in trying to resolve this apparent discrepancy:

1. The sampling procedure was given more weight towards the dominant trees, which were representing 35% and 42.5% for the control and fertilized samples of trees respectively.
2. That the greatest change in form factor occurred in the dominant class of the trees (Chapter 6), and
3. That the ring width contour diagrams showed the greatest response in the upper half of the tree stem and in particular in the dominant and co-dominant classes.

It might therefore be concluded that principal component analysis gives a better indication of the trees' response than relative stem curves, since the first principal component reflects the greater influence of the dominant trees.

Similar results for trees responding to fertilization by changing their form factors have been reported by Woolons and Will (1975) and Whyte and Mead (1977) for Pinus radiata young (13 years old) and mature (40 years old), stands in New Zealand. However, Miller and Cooper (1973) examining the responses of Pinus nigra to fertilization reported changes in basal area, height and volume, but no changes in form factor.

Miller and Cooper (1973) advanced the theory that fertilization response might be better described if it is considered "as a simple analogy of an accelaration in time". Miller (1981) also stated that:

"any parameter that changes with the developmental stage will appear to be altered by fertilizer so long as the treated and untreated trees are compared simply on an equal age basis. Thus as form factor of many species increases over the years prior to maximum current annual increment, fertilization might appear to alter tree shape".

Following this theory form factor of trees may change - since treated trees through fertilization "advance" in age as compared with the controls - provided that they have not yet reached the years of maximum current annual increment. Based on this argument, changes in the stem form of the trees in this study might be explained.

The duration of the fertilization response in this study was four years and in the fifth year the response seemed to decline. This might be attributed to the fact that the increased space that was provided to the trees by the thinning at the beginning of the experiment started decreasing and competition initiated again in the stand. According to Woolons and Will (1975) dealing with response to fertilization in Pinus radiata: "if fertilizers are applied more than three years after thinning then response is inconsistent or transient; no response at all has been detected in unthinned stands".

Therefore a thinning prior to fertilization might provide trees with the proper conditions for a response to fertilization.

The trees of this experiment responded in the year that fertilizers were applied and similar responses have been reported by Brix and Ebell (1969), Brix (1976) and White (1956). The maximum of the response for basal area and height occurred in the second year following fertilization (Brix and Ebell, 1969; Brix,1976).

In estimating responses caused by fertilization increment seems to be a more sensitive variable for the estimation of the response and comparisons between control and fertilized trees, than cumulated results and in particular when volume responses are going to be estimated. As we have seen in this study, comparisons of the volume-basal area lines between control and fertilized trees did not show any statistically significant difference in terms of slope or regression coefficient, denoting that an overall regression could be used for all the trees. When volume increment was regressed with initial BA comparison of the regressions between control and fertilized trees, showed statistically significant differences between control and fertilized trees, and the need for two separate equations for the volume increment estimation.

In the case of estimating volume responses on an area basis the results of this study indicated the need for the inclusion of a form measurement - such as form factor - to be included for the volume estimation of the trees, so that responses to fertilization would be estimated more accurately.

Trees assuming higher positions in the canopy were found to respond more to the treatment. Hamilton (1969) found that for dominant Sitka spruce trees, growth initiation appears earlier and cessation later. This fact, coupled with the bigger vigorous and greater light interception of the crowns, might explain their higher response to fertilization.

9.3 SUGGESTIONS FOR FURTHER RESEARCH

The results of this study presenting evidence of form changes following fertilization stimulate the research in different fields. In mensuration and management to search for some other models of relating volume with BA and perhaps another diameter higher up the stem to account for form changes so that better volume results could be obtained for the estimation of the response on an area basis.

In tree physiology to examine and give explanations about the redistribution of growth following the application of fertilizers and the resulting changes on the tree stem.

In silviculture there appears the need for more experimentation, using different amounts of fertilizers, so that the response surface could be estimated and the right amount and combinations of fertilizers for each particular situation to be estimated. Simulation models could be developed incorporating parameters to account for changes in form, at different levels of fertilization, could be developed so that decisions could be taken about the out turn of saw-logs following fertilization.

The influence of the thinning regime and intensity before the application of fertilizers needs to be examined, also because evidence appears from other experiments that application of thinning before fertilization may yield a better response, and also because in some cases the absence of thinning might be the reason for no response (Woolons and Will, 1975). Combination of thinning and fertilization might be proved a valuable tool in silviculture and management for allocating the higher rate of increment over the stem of the better trees in the crop for early wood production. Finally the effects of fertilization on the stem characteristics of the trees need to be measured for longer periods so that conclusions could be drawn about their future development.

Finally, it could be said that most of the objectives set for this study were obtained although some more analyses could have been carried out (e.g. bark comparisons, crown length comparisons).

9.4 CONCLUSIONS

Evidence presented in this study suggests that following PK fertilization the stem form of pole stage Sitka spruce trees changed. This might be attributed to the increased leaf area of the trees caused by the application of fertilizers which in turn probably caused an improvement in the photosynthetic capacity of the trees.

- The need for a form measurement to be included for volume estimation arises when volume responses are going to be estimated on an area basis.
- Increment is a more sensitive variable in unmasking responses to fertilization than cumulative results.
- The model for diameter prediction, which was developed in this study, using Principal Component Analysis, gave good results.
- Use of relative diameters for the stem curve description, in order to examine the influence of treatment on stem form, gave good results.
- Trees responded to treatment in accordance with their position in the stand canopy.
- Trees responded to fertilization in the year of application.
- The duration of the response, in this experiment, was four years and in the fifth year dropped.

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* DEMAERSCHALK and KOZAK (1977) : see page 192.

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APPENDIX 1

Example of recording the ring width measurements and test of the measuring device used for the ring width measurements

	A r_1	A r_2	A r_3	A r_4	Disc 1 r_1	Disc 1 r_2	Disc 1 r_3	Disc 1 r_4	Disc 2 r_1	Disc 2 r_2	Disc 2 r_3	Disc 2 r_4
width	4.62	4.66	4.64	4.21	3.59	4.01	3.95	2.47	3.53	3.70	3.19	3.56
1978	2.86	0.92	2.42	1.45	2.23	1.18	2.19	1.01	1.83	1.84	1.19	1.95
77	2.14	0.70	1.82	0.92	1.42	0.93	1.49	0.99	0.87	1.23	1.10	1.28
76	2.05	0.74	1.96	0.86	2.00	0.83	1.25	0.86	1.32	0.95	1.02	1.26
75	2.27	1.13	2.21	1.10	2.03	2.04	1.25	0.95	1.55	1.23	1.53	1.86
74	2.75	1.84	2.54	1.21	3.01	2.69	1.49	0.99	2.22	1.56	1.41	1.90
73	2.46	1.95	2.26	1.00	2.32	1.26	1.53	0.99	1.73	1.46	1.16	1.65
72	2.26	1.72	2.12	1.03	2.12	0.98	1.77	1.17	1.23	1.56	1.18	1.95
71	2.40	1.98	1.88	0.97	2.36	0.97	1.17	1.10	1.21	1.03	0.92	1.64
70	2.74	1.92	1.21	0.77	1.48	0.43	0.76	0.78	1.15	0.69	0.67	1.40
69	3.78	2.76	2.29	1.17	1.80	0.61	1.36	1.32	1.83	1.19	1.15	1.38
68	2.19	1.78	1.62	1.49	2.00	1.11	1.80	1.17	1.99	1.77	1.46	1.28
67	2.03	1.27	2.17	1.54	1.90	0.99	1.80	1.18	1.89	1.83	1.77	1.48
66	2.19	1.01	1.96	1.07	2.30	1.05	1.70	1.56	2.32	1.79	1.66	1.58
65	3.48	1.52	2.50	1.57	3.31	1.32	1.44	1.87	3.02	2.95	2.10	2.20
64	3.79	1.67	3.20	1.73	3.17	1.92	2.69	1.73	2.86	2.83	2.60	2.65
63	3.25	1.27	3.12	1.69	2.94	2.09	2.27	1.74	2.98	3.03	2.91	2.44
62	4.29	2.71	4.42	2.24	3.49	3.29	3.67	2.58	3.15	3.92	3.67	2.58
61	4.58	2.48	4.07	2.13	3.17	2.99	3.84	2.75	3.20	3.29	3.42	2.91
60	4.72	2.55	4.25	2.88	3.88	3.21	4.42	3.23	3.76	3.99	4.19	4.20
59	5.73	4.54	5.75	4.49	4.61	4.21	4.69	4.33	5.13	4.47	4.94	4.71
58	7.14	7.28	7.98	6.44	6.13	7.41	6.93	6.81	8.36	6.38	7.78	6.19
57	7.01	5.66	6.93	5.47	6.62	6.95	7.34	7.26	7.99	6.32	7.67	7.43
56	5.34	5.15	5.91	4.24	6.33	5.99	6.07	5.77	6.22	5.49	5.33	6.46
55	5.58	6.07	5.80	5.12	6.64	5.74	4.70	6.31	5.26	5.34	5.05	5.84
54	6.55	8.25	7.99	7.23	6.97	6.26	7.60	6.72				
53	4.65	5.70	5.68	4.50	4.12	3.67	4.50	3.38				
52	3.00	3.21	2.99	2.65								
Pith	3.46	3.81	3.47	4.14	2.24	1.90	2.11	2.09	4.70	4.77	4.48	5.00

Fig. 1. Example of recording the ring width measurements

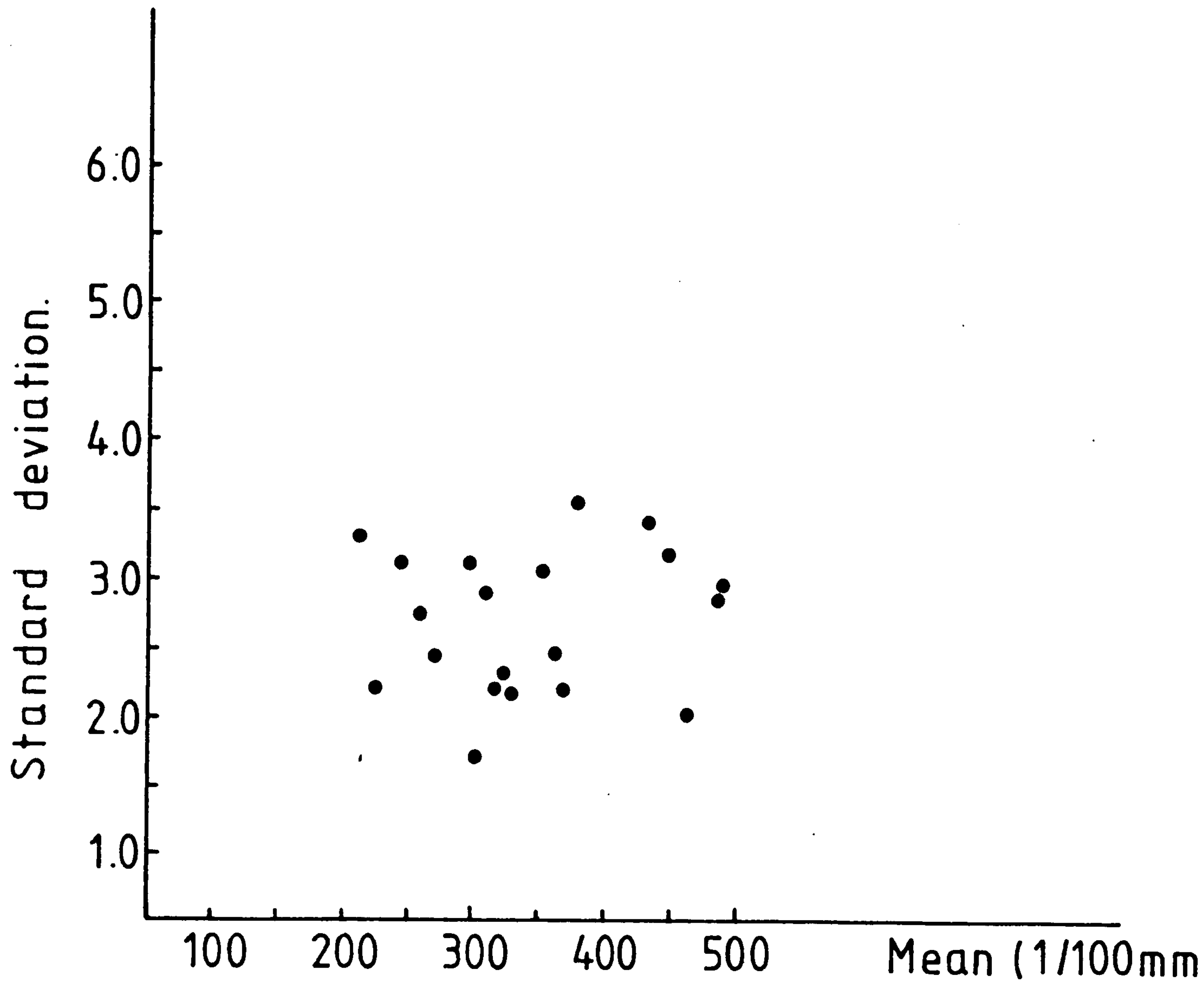

Fig. 2 Plotting of the mean ring measurements against their standard deviation.

APPENDIX 2

DBH frequency distribution of the control and fertilized trees

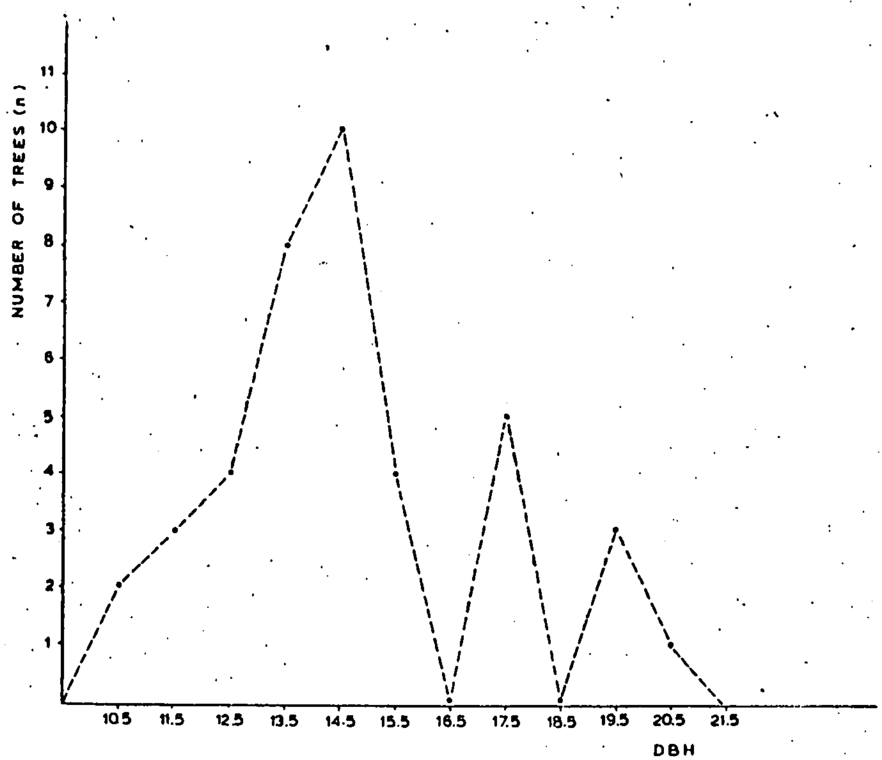

Fig. 1 Dbh frequency distribution of the 40 sampled control trees (1971)

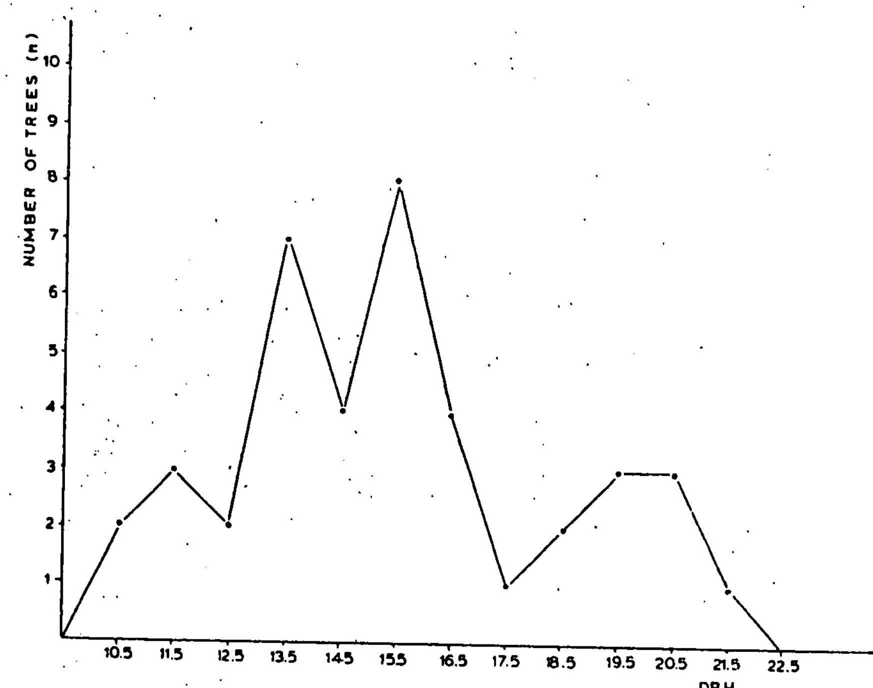

Fig. 2 Dbh frequency distribution of the 40 sampled control trees (1975)

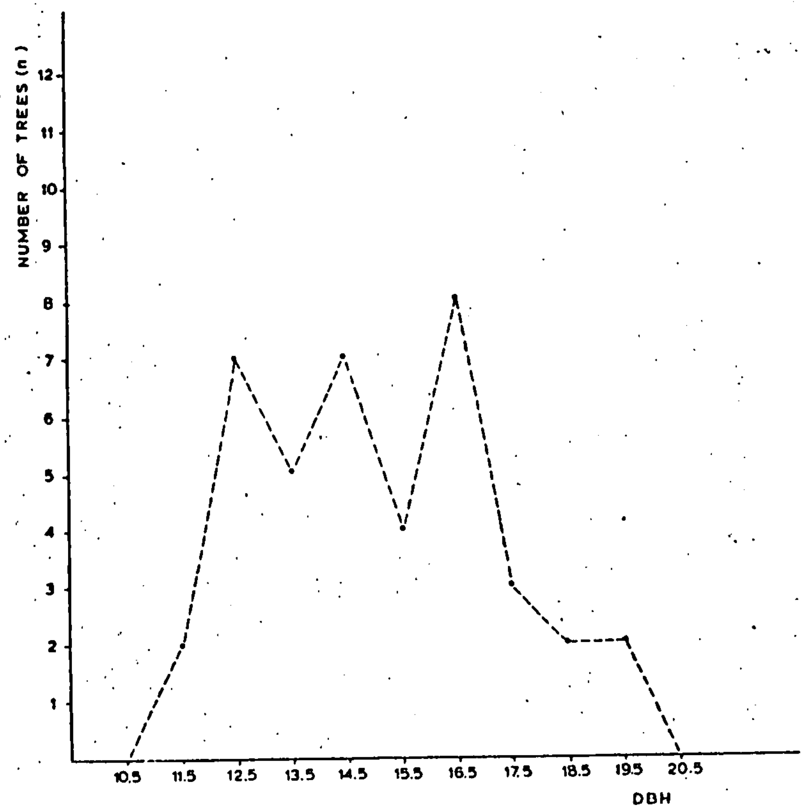

Fig. 3 Dbh frequency distribution of the 40 sampled fertilized trees (1971)

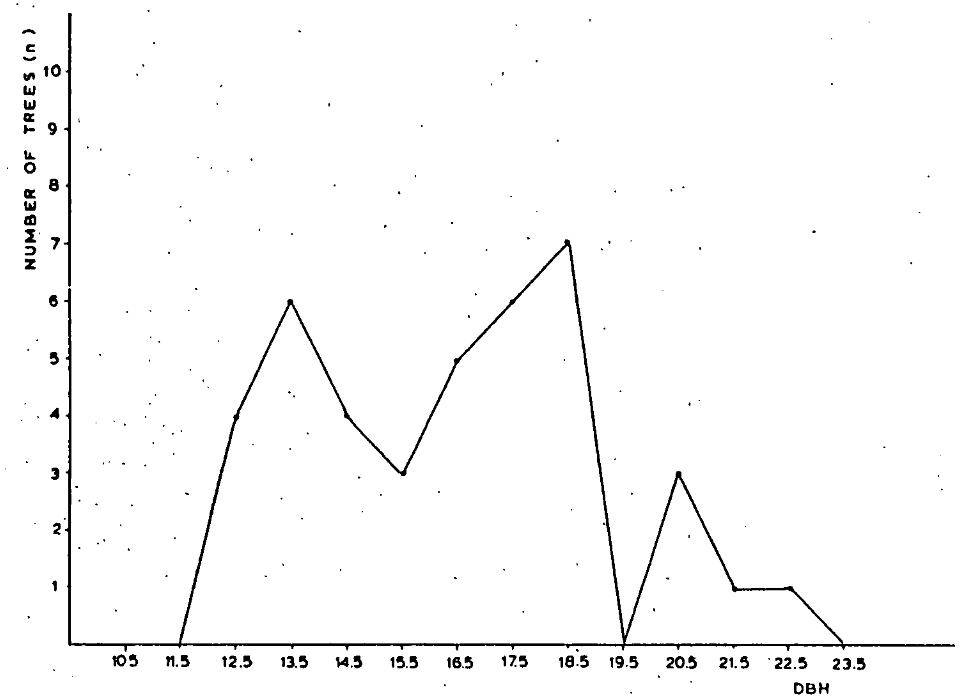

Fig.4 Dbh frequency distribution of the 40 sampled fertilized trees (1975)

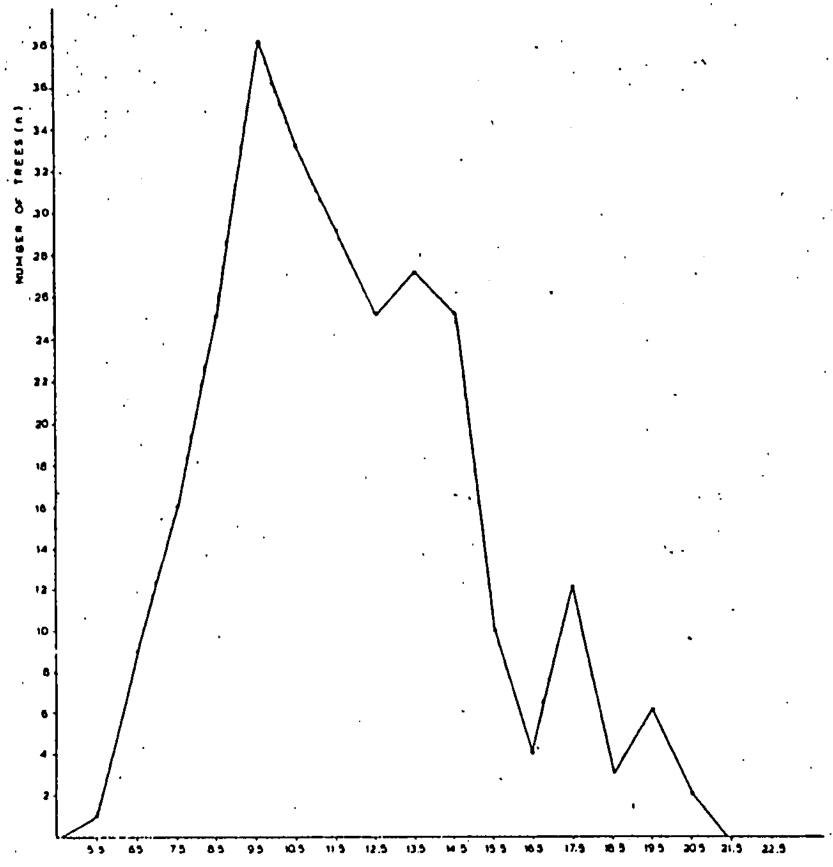

Fig. 5 Dbh frequency distribution of all the control trees of all plots (1971)

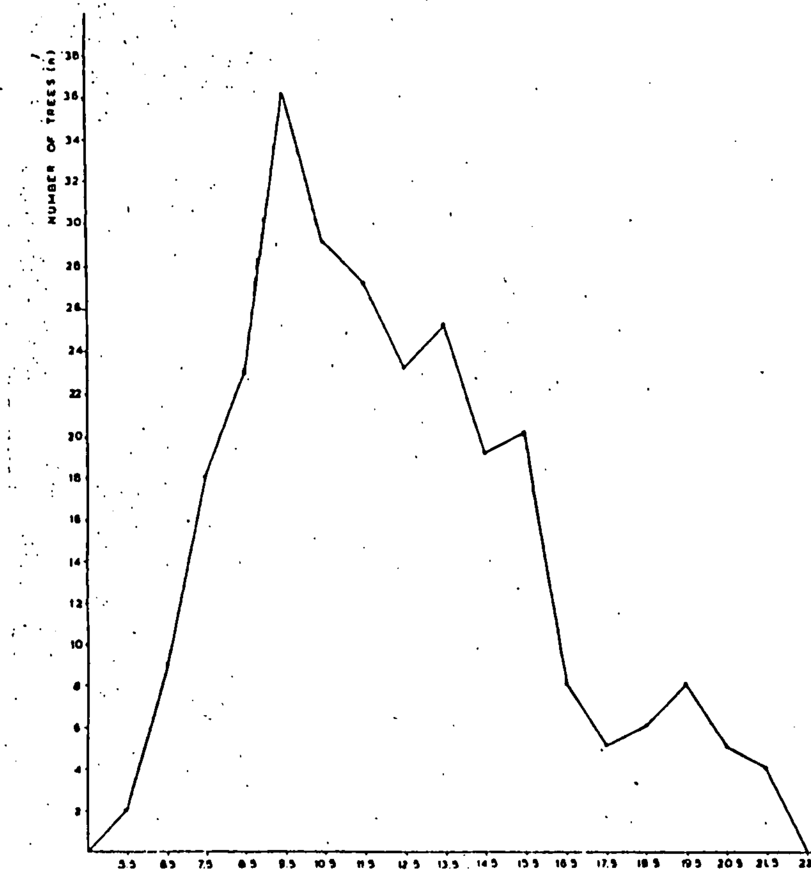

Fig. 6 Dbh frequency distribution of all the control trees of all plots (1975)

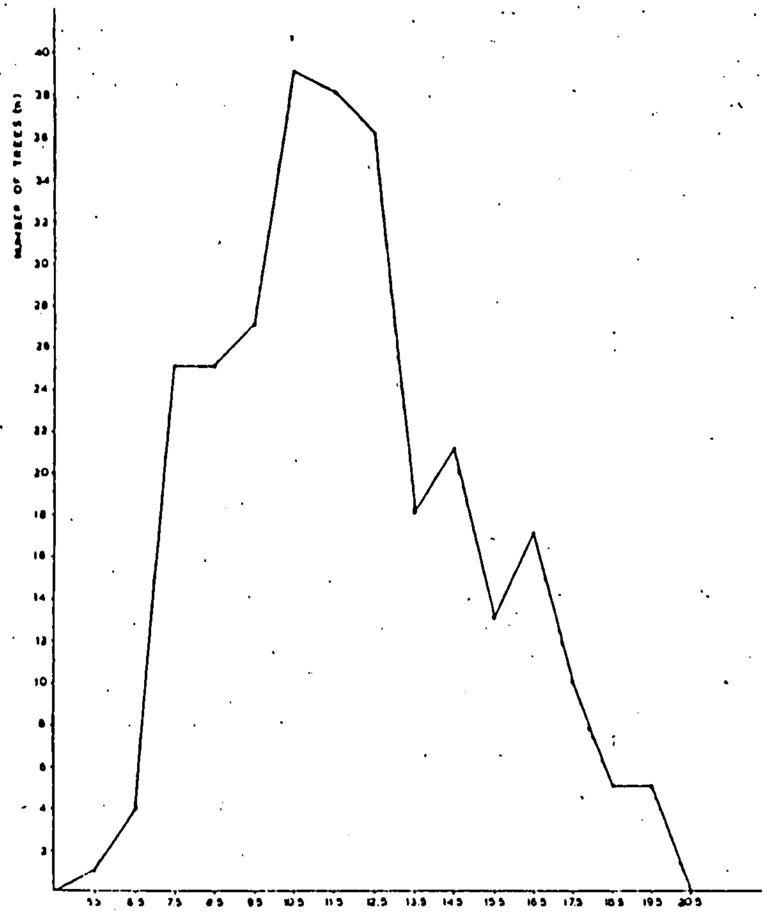

Fig. 7 Dbh frequency distribution of all the fertilized trees of all plots (1971)

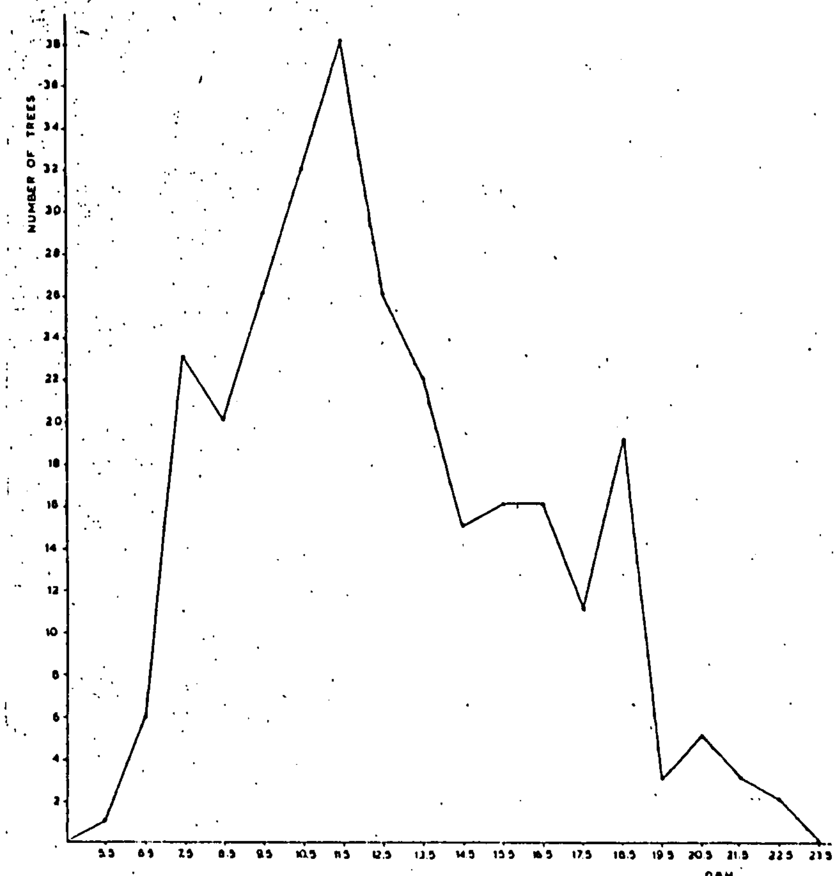

Fig. 8 Dbh frequency distribution of all the fertilized trees of all plots (1975)

APPENDIX 3

Tables of statistical analysis

* The lack of significant figures in some of the following tables in the "Sum of squares" and "mean square" columns is due to the limitations of the fixed format output of SPSS package. This does not affect the validity of the calculations of the variance ratio.

TABLE 1

ANALYSIS OF VARIANCE OF BASAL AREA IN 1970

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.000	3	0.000	3.905*
TREATMENTS	0.000	1	0.000	0.035
EXPLAINED	0.000	7	0.000	2.060
RESIDUAL	0.001	72	0.000	
TOTAL	0.002	79	0.000	

*ss at $p < 0.05$

$F_{TAB} = 2.74$ at $p = 0.05$ for 3 and 70 d.f.

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE 2

ANALYSIS OF VARIANCE OF BASAL AREA IN 1975 BETWEEN CONTROL AND FERTILIZED TREES

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.000	3	0.000	3.315*
TREATMENTS	0.000	1	0.000	2.057
EXPLAINED	0.001	7	0.000	2.028
RESIDUAL	0.003	72	0.000	
TOTAL	0.003	79	0.000	

*Ss at $p < 0.05$

$F_{TAB} = 2.74$ at $p=0.05$ with 3 and 70 d.f.

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE 3

ANALYSIS OF COVARIANCE OF BASAL AREA IN 1975 WITH COVARIATE BASAL AREA 1970

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.000	3	0.000	0.512
TREATMENTS	0.000	1	0.000	26.820**
COVARIATES	0.003	1	0.003	1190.689
EXPLAINED	0.003	5	0.001	
RESIDUAL	0.000	74	0.000	
TOTAL	0.003	79	0.000	

**ss at $p < 0.01$

$F_{TAB} = 7.01$ at $p = 0.01$ with 1 and 70 d.f.

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE 4

ANALYSIS OF VARIANCE OF BASAL AREA INCREMENT DURING 1970-75

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.000	3	0.000	1.481
TREATMENTS	0.000	1	0.000	13.633**
EXPLAINED	0.000	7	0.000	2.760
RESIDUAL	0.000	72	0.000	
TOTAL	0.000	79	0.000	

**ss at $p < 0.01$

F_{TAB}= 7.01 at p=0.01 with 1 and 70 d.f.

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE 5

ANALYSIS OF VARIANCE OF CROSS-SECTIONAL INCREMENT AT HALF TOTAL HEIGHT DURING 1970–75

Source of Variation	Sum of Squares	D.F	Mean Square	Variance Ratio F
Blocks	0.0000148	3	0.00000493	0.861
Treatments	0.0000405	1	0.0000405	15.184 **
Error	0.0001923	72	0.00000267	
Total		79		

** SS at $p < 0.01$

$F_{Tab} = 7.02$ at $p = 0.01$, with 1 and 70 d.f

TABLE 6

REGRESSION OF BASAL AREA INCREMENT 1970-75 WITH BASAL AREA IN 1970.

$\Delta G = -0.676 + 0.292\ G$ CONTROL TREES (n=40)

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.85	REGRESSION	0.00009	1	0.00009	107.535**
SE=0.0009	RESIDUAL	0.00003	38	0.00000	

ΔG=Basal area increment during 1970-75

G=Basal area in 1970

**ss at $p < 0.01$

TABLE 7

REGRESSION OF BASAL AREA INCREMENT 1970-75 WITH BASAL AREA IN 1970

$\Delta G=-0.689 + 0.418\ G$ FERTILIZED TREES (n=40)

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.70	REGRESSION	0.00013	1	0.00013	34.78**
SE=0.0191	RESIDUAL	0.00014	38	0.00000	

ΔG=Basal area increment during 1970-75

G =Basal area in 1970

** ss at p 0.01

TABLE 8

ANALYSIS OF VARIANCE OF HEIGHT IN 1975 BETWEEN CONTROL AND FERTILIZED TREES

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	51.043	3	17.014	8.947**
TREATMENTS	13.857	1	13.857	9.382**
EXPLAINED	64.901	4	16.225	7.641
RESIDUAL	136.014	75	1.814	8.947
TOTAL	200.914	79	2.543	

**ss at $p<0.01$

$F_{TAB} = 7.01$ at $p=0.01$ for 1 and 70 d.f.

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE 9

ANALYSIS OF COVARIANCE OF HEIGHT 1975 WITH COVARIATE HEIGHT 1970

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.749	3	0.250	0.972
TREATMENTS	11.935	1	11.935	46.431**
COVARIATES	116.932	1	116.992	455.146
EXPLAINED	181.893	5	36.379	141.527
RESIDUAL	19.021	74	0.257	
TOTAL	200.914	79	2.543	

** ss at $p < 0.01$

$F_{TAB}=7.01$ at $p=0.01$, with 1 and 70 d.f.

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE 10

REGRESSION OF HEIGHT 1975 WITH HEIGHT 1970 (CONTROL TREES n=40)

$HE_{75}=1.236 + 1.064\ HE_{70}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.94	REGRESSION	92.077	1	92.077	314.143**
SE=0.541	RESIDUAL	11.138	38	0.293	

HE_{75}= Height in 1975

HE_{70}= Height in 1970

**ss at $p < 0.01$

TABLE 11

REGRESSION OF HEIGHT 1975 WITH HEIGHT 1970 (FERTILIZED TREES n=40)

$$HE_{75} = 3.061 + 0.984\ HE_{70}$$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R= 0.94	REGRESSION	77.550	1	77.550	330.238**
SE= 0.485	RESIDUAL	8.923	38	0.2348	

HE_{75}= Height in 1975

HE_{70}= Height in 1970

**ss at $p < 0.01$

TABLE 12

ANALYSIS OF VARIANCE OF FORM FACTOR CHANGE

DURING 1970-75

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.003	3	0.001	1.797
TREATMENTS	0.004	1	0.004	7.189**
EXPLAINED	0.009	7	0.001	2.103
RESIDUAL	0.045	72	0.001	
TOTAL	0.054	79	0.001	

**ss at $p < 0.01$

$F_{TAB}=7.01$ for 1 and 72 d.f. at $p=0.01$

Number of trees used in the analysis: 40 control and 40 fertilized

TABLE13

ANALYSIS OF VARIANCE OF FORM FACTOR CHANGE

DURING 1965-70

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.005	3	0.002	2.251
TREATMENTS	0.000	1	0.000	0.038 NS
EXPLAINED	0.005	7	0.001	1.022
RESIDUAL	0.054	72	0.001	
TOTAL	0.059	79		

F_{TAB}=2.74 with 1 and 70 d.f. at p=0.005

NS= No ss differences at $p \leqslant 0.05$

Number of trees used in the analysis: 40 control to be and 40 fertilized to be

TABLE 14

ANALYSIS OF VARIANCE OF VOLUME IN 1975

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARES	VARIANCE RATIO F
BLOCKS	0.012	3	0.004	3.793*
TREATMENTS	0.000	1	0.000	0.213
EXPLAINED	0.018	7	0.003	2.582
RESIDUAL	0.073	72	0.001	
TOTAL	0.091	79	0.001	

*ss at $p < 0.05$

$F_{TAB} = 2.74$ at $p=0.05$ for 3 and 72 d.f.

Number of trees used: 40 control and 40 fertilized

TABLE 15

ANALYSIS OF COVARIANCE OF VOLUME 1975 WITH COVARIATE VOLUME 1970

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.000	3	0.000	0.458
TREATMENTS	0.011	1	0.011	524.747 **
COVARIATES	0.193	1	0.193	8887.625
EXPLAINED	0.255	8	0.032	1465.902
RESIDUAL	0.002	71	0.000	
TOTAL	0.257	79	0.003	

**ss at $p < 0.01$

$F_{TAB}=7.01$ at $p=0.01$, with 1 and 70 d.f.

Number of trees used: 40 control and 40 fertilized

TABLE 16

ANALYSIS OF VOLUME INCREMENT DURING 1970-75

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
BLOCKS	0.005	3	0.002	2.455
TREATMENTS	0.011	1	0.011	15.926**
EXPLAINED	0.016	7	0.002	3.443
RESIDUAL	0.049	72	0.001	
TOTAL	0.065	79	0.001	

**ss at $p < 0.01$

$F_{TAB} = 7.01$ at $p = 0.01$ for 1 and 70 d.f.

Number of trees used: 40 control and 40 fertilized

TABLE 17

OVERALL REGRESSION OF VOLUME INCREMENT DURING 1970-75 WITH BASAL AREA 1970

$$VI = -0.0153 + 4.642\ G_{70}$$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R= 0.762	REGRESSION	0.0379	1	0.0379	108.195**
SE= 0.018	RESIDUAL	0.0273	78	0.0003	

**ss at $p < 0.01$

VI= Volume Increment during 1970-75

G_{70}= Basal area in 1970

TABLE 18

REGRESSION OF VOLUME INCREMENT DURING 1970-75 WITH BASAL AREA 1970

A. CONTROL TREES $VI = -0.0123 + 3.631\ G_{70}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R= 0.895	REGRESSION	0.0136	1	0.0136	153.729 **
SE=0.009	RESIDUAL	0.0033	38	0.00009	

B. FERTILIZED TREES $VI = -0.0229 + 5.967\ G_{70}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R= 0.831	REGRESSION	0.0258	1	0.0258	84.481 **
SE=0.017	RESIDUAL	0.0116	38	0.0003	

** ss at $p < 0.001$

VI= Volume increment

G_{70}= Basal area 1970

TABLE 19

COMPARISON OF THE REGRESSIONS OF VOLUME INCREMENT 1970-75 WITH BASAL AREA 1970 BETWEEN CONTROL AND FERTILIZED ANOVAR OF DIFFERENCES

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
COMBINED REGRESSION	0.03754	1	0.03754	178.224
BETWEEN COEFFICIENTS	0.00239	1	0.00239	11.391**
BETWEEN CONSTANTS	0.00985	1	0.00985	46.799**
RESIDUAL	0.01600	76	0.00021	
TOTAL	0.06580	79		

**ss differences at $p < 0.01$

$F_{TAB} = 6.96$ for 1 and 80 d.f. at $p = 0.01$

TABLE 20

REGRESSION OF VOLUME IN 1975 WITH VOLUME IN 1970

A. CONTROL TREES $V_{75} = 0.0024 + 1.496\ V_{70}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.986	REGRESSION	0.12271	1	0.1227	1328.80**
SE=0.0096	RESIDUAL	0.0035	38	0.00009	

B. FERTILIZED TREES $V_{75} = -0.0025 + 1.789\ V_{70}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.950	REGRESSION	0.1241	1	0.1241	355.32 **
SE=0.0186	RESIDUAL	0.0132	38	0.0003	

**ss at $p < 0.01$

V_{75}= volume in 1975

V_{70}= volume in 1970

TABLE 21

COMPARISONS OF THE REGRESSIONS OF VOLUME 1975 WITH VOLUME 1970 BETWEEN CONTROL AND FERTILIZED TREES ANOVAR OF DIFFERENCES

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
COMBINED REGRESSION	0.24337	1	0.24337	1101.853
BETWEEN COEFFICIENTS	0.00195	1	0.00195	8.849**
BETWEEN CONSTANTS	0.01141	1	0.01141	51.660**
RESIDUAL	0.01678	76	0.00022	
TOTAL	0.27352	79		

**ss differences at $p < 0.01$

$F_{TAB}=6.96$ for 1 and 80 d.f. at $p=0.01$

TABLE 22

REGRESSION OF VOLUME IN 1975 WITH BASAL AREA IN 1975

A. CONTROL TREES $V_{75} = -0.0217 + 8.197\ G_{75}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.967	REGRESSION	0.1180	1	0.1180	547.84**
SE=0.014	RESIDUAL	0.0081	38	0.0002	

B. FERTILIZED TREES $V_{75} = -0.0320 + 9.014\ G_{75}$

	ANOVA				
	SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
R=0.971	REGRESSION	0.1298	1	0.1298	647.92**
SE=0.014	RESIDUAL	0.0076	38	0.0002	

**ss at $p < 0.01$

V_{75}=Volume in 1975

G_{70}=Basal area in 1975

TABLE 23

COMPARISON OF THE REGRESSIONS OF VOLUME IN 1975 WITH BASAL AREA 1975 BETWEEN CONTROL AND FERTILIZED ANOVAR OF DIFFERENCES

SOURCE OF VARIATION	SUM OF SQUARES	D.F.	MEAN SQUARE	VARIANCE RATIO F
COMBINED REGRESSION	0.25668	1	0.25668	1,231.861
BETWEEN COEFFICIENTS	0.00053	1	0.00053	2.589 N.S
BETWEEN CONSTANTS	0.00046	1	0.00046	2.241 N.S
RESIDUAL	0.01583	76	0.00021	
TOTAL	0.27352	79		

N.S. No statistically significant differences at $p < 0.05$

$F_{TAB} = 3.96$ for 1 and 80 d.f. at $p = 0.05$

TABLE 24

METHOD 1 - REGRESSION ANOVA - V=a+b G

CONTROL TREES (n=40)

SOURCE	SS	d.f.	MS	VARIANCE RATIO F	F TABLE
REGRESSION	0.11875873	1	0.11875873	605.018**	7.35
RESIDUAL	0.00745899	38	0.000196289		
TOTAL	0.1262176	39			

FERTILIZED TREES (n=40)

SOURCE	SS	d.f.	MS	VARIANCE RATIO F	F TABLE
REGRESSION	0.13081599	1	0.13081599	751.036**	7.35
RESIDUAL	0.06618866	38	0.000174180		
TOTAL	0.19700465	39			

NOTES:

V=Volume underbark estimated from stem analysis in 1975

G=overbark basal area in 1975

** statistically significant at $p < 0.01$

TABLE 25

COMPARISON OF THE TWO REGRESSIONS FOR METHOD 1

ANOVAR OF DIFFERENCES

SOURCE	SS	d.f.	MS	VR	TABLE F
COMBINED REGRESSION	0.258964	1	0.258964	1398.03	3.9
BETWEEN COEFFICIENTS	0.000338	1	0.000338	1.8258	N.S
BETWEEN CONSTANTS	0.000148	1	0.000148	0.8021	N.S
RESIDUAL	0.014077	76	0.000185		
TOTAL	0.273529	79			

NS No differences at $p \leq 0.05$

TABLE 26

METHOD 2 - REGRESSION ANOVA - VI=a+b G_{71}

CONTROL TREES (n=40)

SOURCE	SS	d.f.	MS	VARIANCE RATIO F	TABLE F
REGRESSION	0.00246202	1	0.00946202	141.70**	7.35
RESIDUAL	0.00253734	38	0.00006677		
TOTAL	0.01199936	39			

FERTILIZED TREES (n=40)

SOURCE	SS	d.f.	MS	VARIANCE RATIO F	TABLE F
REGRESSION	0.01927313	1	0.01927313	113.39**	7.35
RESIDUAL	0.00645858	38	0.00169962		
TOTAL	0.02573171	39			

NOTES:

VI=Volume increment during 1971-75 estimated from stem analysis data

G=Basal area estimated overbark in 1971

**statistically significant at $p < 0.01$

TABLE 27

COMPARISON OF THE REGRESSIONS FOR METHOD 2 $\Delta V = a + b\, G_{71}$

ANOVAR OF DIFFERENCES

SOURCE	SS	d.f.	MS	VR	TABLE F
COMBINED REGRESSION	0.0286539	1	0.0286539	242.07**	
BETWEEN COEFFICIENTS	0.0016955	1	0.0016955	14.324**	3.9
BETWEEN CONSTANTS	0.0056284	1	0.0056284	47.550**	
RESIDUAL	0.0089955	76	0.000118367		
TOTAL	0.044973	79			

**significant at $p < 0.01$